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# Matrices - what is a matrix ?

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This leaflet will explain what is meant by a **matrix** and the notation we use to describe matrices. We will also look at some special types of matrix.

A **matrix** is a rectangular pattern of numbers - we usually enclose the numbers with brackets. So, for example, the following are all matrices.

$$\begin{pmatrix} 4 & -1 \\ 13 & 9 \end{pmatrix} \qquad \begin{pmatrix} 12 & 3 & 0 & 4 \end{pmatrix} \qquad \begin{pmatrix} 7 & 1 \\ -3 & 2 \\ 4 & 4 \end{pmatrix} \qquad \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$$

Note that in each case we have a rectangular pattern of numbers. These numbers can be any numbers we choose - positive, negative, zero, fractions, decimals, and so on. To refer briefly to a specific matrix we might label it, usually with a capital letter, so we might write

$$A = \begin{pmatrix} 4 & -1 \\ 13 & 9 \end{pmatrix} \qquad B = \begin{pmatrix} 12 & 3 & 0 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 7 & 1 \\ -3 & 2 \\ 4 & 4 \end{pmatrix} \qquad D = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0.7 \end{pmatrix}$$

Clearly, all these matrices have different sizes. When we want to refer to the size of a matrix we state its number of **rows** and number of **columns**, in that order. Matrix $A$ has two rows and two columns; we write that it is a $2 \times 2$ matrix and say that it is a 'two by two' matrix.

Similarly we observe $B$ is $1 \times 4$, $C$ is $3 \times 2$ and $D$ is $3 \times 3$.

Each number in a matrix is referred to as an **element** of the matrix. If we want to write down a general matrix $A$ with $m$ rows and $n$ columns we write

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

Here, the symbol $a_{21}$ represents the element in the second row, first column, and so on. Generally, $a_{ij}$ represents the element in the $i$th row and $j$th column.

## Some special types of matrix

Some types of matrix occur quite frequently, have special properties or are particularly important. We give these matrices special names.

A **square** matrix, as the name suggests, has the same number of rows as columns. So the matrices $A$ and $D$ above are square.

A **diagonal** matrix is a square matrix with zeros everywhere except possibly on the diagonal which runs from the top left to the bottom right. This diagonal is called the **leading diagonal**. Matrix $D$

is a diagonal matrix. Here are some more diagonal matrices:

$$E = \begin{pmatrix} 4 & 0 \\ 0 & -2 \end{pmatrix} \qquad F = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -7 \end{pmatrix} \qquad G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Note that whereas all the non-diagonal elements are zero, the elements on the leading diagonal can be any number including zero.

An **identity** matrix, sometimes called a **unit matrix**, is a diagonal matrix with all its diagonal elements equal to 1. The following are identity matrices.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The symbol $I$ is usually reserved for labelling identity matrices.

If we are dealing with several identity matrices at the same time, and because we usually use the letter $I$ to denote an identity matrix, we might use a subscript to indicate the size of the particular identity matrix we are discussing. So we might write

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity matrices have a special and very important property. We shall see in a later leaflet, when we consider multiplication of matrices, that multiplying a matrix by an identity matrix, leaves that matrix unchanged.

The next leaflet in this series will look at what is meant by a symmetric matrix and the transpose of a matrix.

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Note that a video tutorial covering the content of this leaflet is available from **sigma**.

 