The z transform

Given a sequence, $f[k]$, $k = 0, 1, 2\ldots$, the (one-sided) z **transform** of $f[k]$, is $F(z)$ defined by

$$F(z) = \mathcal{Z}\{f[k]\} = \sum_{k=0}^{\infty} f[k]z^{-k}.$$

sequence $f[k]$	z transform $F(z)$
$\delta[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$	1
$u[k] = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$	$\frac{z}{z-1}$
k	$\frac{z}{(z-1)^2}$
e^{-ak}	$\frac{z}{z-\mathrm{e}^{-a}}$
a^k	$\frac{z}{z-a}$
ka^k	$\frac{az}{(z-a)^2}$
k^2	$\frac{z(z+1)}{(z-1)^3}$
$\sin ak$	$\frac{z\sin a}{z^2-2z\cos a+1}$
$\cos ak$	$\frac{z(z-\cos a)}{z^2-2z\cos a+1}$
$\mathrm{e}^{-ak}\sin bk$	$\frac{z\mathrm{e}^{-a}\sin b}{z^2-2z\mathrm{e}^{-a}\cos b+\mathrm{e}^{-2a}}$
$\mathrm{e}^{-ak}\cos bk$	$\frac{z^2-z\mathrm{e}^{-a}\cos b}{z^2-2z\mathrm{e}^{-a}\cos b+\mathrm{e}^{-2a}}$
$\mathrm{e}^{-bk}f[k]$	$F(\mathrm{e}^{b}z)$
$kf[k]$	$-z\frac{\mathrm{d}}{\mathrm{d}z}F(z)$

Linearity:

If $f[k]$ and $g[k]$ are two sequences and c is a constant

$$\mathcal{Z}\{f[k]+g[k]\} = \mathcal{Z}\{f[k]\} + \mathcal{Z}\{g[k]\}.$$

$$\mathcal{Z}\{cf[k]\} = c\mathcal{Z}\{f[k]\}.$$

First shift theorem:

$$\mathcal{Z}\{f[k+1]\} = zF(z) - zf[0].$$

$$\mathcal{Z}\{f[k+2]\} = z^2F(z) - z^2f[0] - zf[1].$$

Second shift theorem:

$$\mathcal{Z}\{f[k-i]u[k-i]\} = z^{-i}F(z), \qquad i = 1, 2, 3 \ldots$$

where $F(z)$ is the z transform of $f[k]$ and $u[k]$ is the unit step sequence.

Convolution:

$$\mathcal{Z}\{f[k] * g[k]\} = F(z)G(z).$$

where

$$f[k] * g[k] = \sum_{m=0}^{k} f[m]g[k-m].$$