Maclaurin & Taylor Series

Maclaurin Series:

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$$

Taylor series (one variable):

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \ldots + \frac{(x-a)^r}{r!}f^{(r)}(a) + \ldots$$

Taylor series (two variables):

For a function $f(x, y)$ of two variables

$$\begin{aligned} f(x,y) &= f(a,b) + \frac{1}{1!}\left((x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right) f(a,b) \\ &\quad + \frac{1}{2!}\left((x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right)^2 f(a,b) + \ldots \\ &\quad + \frac{1}{r!}\left((x-a)\frac{\partial}{\partial x} + (y-b)\frac{\partial}{\partial y}\right)^r f(a,b) + \ldots \end{aligned}$$

Stationary points in two variables:

For $z = f(x, y)$, stationary points (a, b) are located by solving

$$\frac{\partial f}{\partial x} = 0 \text{ and } \frac{\partial f}{\partial y} = 0.$$

Define $\Delta = \dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial^2 f}{\partial y^2} - \left(\dfrac{\partial^2 f}{\partial x \partial y}\right)^2$ at (a, b).

The type of stationary point is given by:

$$\Delta < 0 \quad \text{saddle point.}$$

$$\Delta > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} > 0 \quad \text{minimum point.}$$

$$\Delta > 0 \text{ and } \frac{\partial^2 f}{\partial x^2} < 0 \quad \text{maximum point.}$$