The Laplace transform

The **Laplace transform** of $f(t)$ is $F(s)$ defined by

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t) \mathrm{d}t.$$

function $f(t), t \geqslant 0$	Laplace transform $F(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sinh bt$	$\frac{b}{s^2-b^2}$
$\cosh bt$	$\frac{s}{s^2-b^2}$
$t \sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t \cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$u(t)$ unit step	$\frac{1}{s}$
$\delta(t)$ impulse function	1
$\delta(t-a)$	e^{-sa}
$f(t)$ periodic	$\frac{\int_0^T e^{-st} f(t) \mathrm{d}t}{1-e^{-sT}}$
$t^n f(t)$	$(-1)^n \frac{\mathrm{d}^n}{\mathrm{d}s^n} F(s)$

Linearity:

$$\mathcal{L}\{f+g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}, \qquad \mathcal{L}\{kf\} = k\mathcal{L}\{f\}.$$

Shift theorems: If $\mathcal{L}\{f(t)\} = F(s)$ then

$$\mathcal{L}\{e^{-at}f(t)\} = F(s+a).$$

$$\mathcal{L}\{u(t-d)f(t-d)\} = e^{-sd}F(s) \qquad d > 0.$$

$u(t)$ is the unit step or Heaviside function.

Laplace transform of derivatives and integrals:

$$\mathcal{L}\{f'\} = sF(s) - f(0).$$

$$\mathcal{L}\{f''\} = s^2F(s) - sf(0) - f'(0).$$

$$\mathcal{L}\left\{\int_0^t f(t)\mathrm{d}t\right\} = \frac{1}{s}F(s).$$

The convolution theorem:

The Laplace transform of $f(t) * g(t)$ is $F(s)G(s)$ where

$$f(t) * g(t) = \int_0^t f(t-\lambda)g(\lambda)\,\mathrm{d}\lambda = g(t) * f(t).$$