Complex Integrals

When evaluating the integrals which follow, the curve C is traversed in an anticlockwise sense.

Cauchy's theorem: If $f(z)$ is analytic within and on a simple closed curve C then $\oint_C f(z)\mathrm{d}z = 0$.

Cauchy's integral formula: If $f(z)$ is analytic within and on a simple closed curve C, and if z_0 is any point within C then

$$\oint_C \frac{f(z)}{z - z_0}\mathrm{d}z = 2\pi j f(z_0).$$

Further

$$\oint_C \frac{f(z)}{(z - z_0)^{n+1}}\mathrm{d}z = \frac{2\pi j}{n!} f^{(n)}(z_0).$$

The residue theorem: If $f(z)$ is analytic within and on a simple closed curve C apart from a finite number of poles inside C, then

$$\oint_C f(z)\mathrm{d}z = 2\pi j \times [\text{ sum of residues of } f(z) \text{ at the poles inside } C].$$

Eigenvalues & Eigenvectors

An **eigenvector** of a square matrix A is a non-zero column vector X such that $AX = \lambda X$ where λ, (a scalar), is the corresponding **eigenvalue**. The eigenvalues are found by solving the **characteristic equation**

$$\det(A - \lambda I) = 0.$$

An $n \times n$ symmetric matrix A with real elements has only real eigenvalues and n independent eigenvectors. The eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal. The **modal matrix** corresponding to the $n \times n$ square matrix A is an $n \times n$ square matrix P whose columns are the eigenvectors of A. If n independent eigenvectors are used to form P then $P^{-1}AP$ is a diagonal matrix in which the diagonal entries are the eigenvalues of A taken in the same order that the eigenvectors were taken to form P.