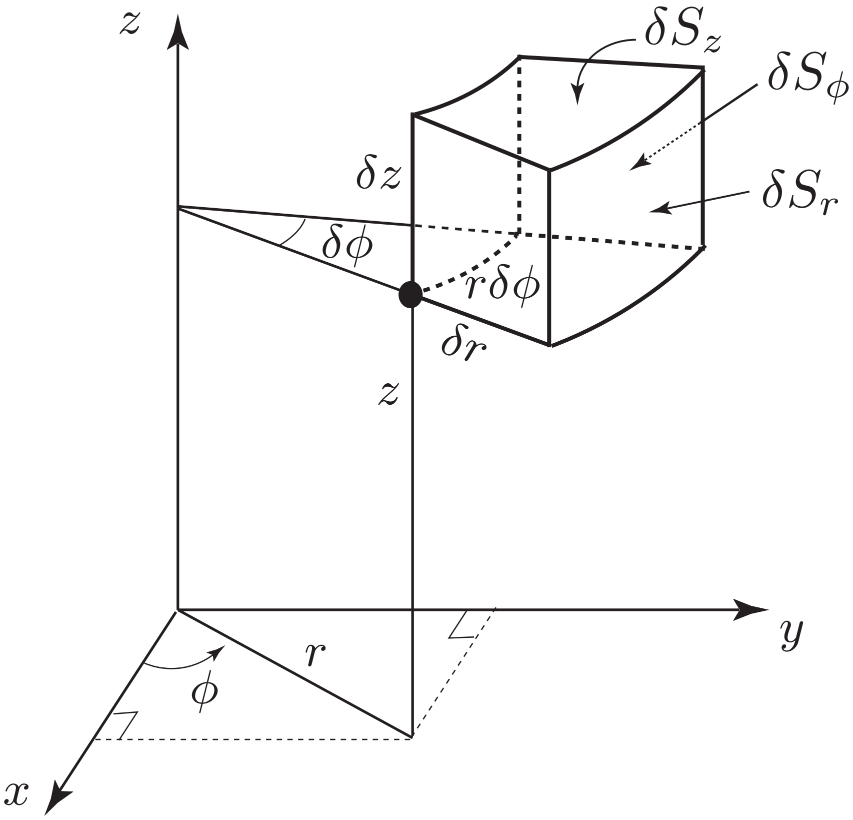
# Cylindrical polar coordinates

The diagram shows cylindrical polar coordinates $(r, \phi, z)$.




$$\left.\begin{array}{c} x = r\cos\phi \\ y = r\sin\phi \\ z = z \end{array}\right\} \qquad \begin{array}{c} r \geq 0 \\ 0 \leq \phi < 2\pi \\ -\infty < z < \infty \end{array}$$

If $\mathbf{v} = v_r\hat{\mathbf{e}}_r + v_\phi\hat{\mathbf{e}}_\phi + v_z\hat{\mathbf{e}}_z$:

$$\nabla\Phi = \frac{\partial\Phi}{\partial r}\hat{\mathbf{e}_r} + \frac{1}{r}\frac{\partial\Phi}{\partial\phi}\hat{\mathbf{e}_\phi} + \frac{\partial\Phi}{\partial z}\hat{\mathbf{e}_z}.$$

$$\nabla\cdot\mathbf{v} = \frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial}{\partial\phi}(v_\phi) + \frac{\partial v_z}{\partial z}.$$

$$\nabla \times \mathbf{v} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{e}}_r & r\hat{\mathbf{e}}_\phi & \hat{\mathbf{e}}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ v_r & rv_\phi & v_z \end{vmatrix}.$$

$$\nabla^2\Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial \phi^2} + \frac{\partial^2\Phi}{\partial z^2}.$$

Volume element: $\delta V = r\,\delta r\,\delta\phi\,\delta z$.

Surface elements:
$\delta S_r = r\,\delta\phi\,\delta z$,
$\delta S_\phi = \delta r\,\delta z$,
$\delta S_z = r\,\delta r\,\delta\phi$.

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