The Fourier transform

The **Fourier transform** of $f(t)$ is $F(\omega)$ defined by

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\mathrm{d}t = F(\omega).$$

The **inverse Fourier transform** is given by

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}\mathrm{d}\omega = f(t).$$

function $f(t)$	Fourier transform $F(\omega)$
$Au(t)e^{-\alpha t},\ \alpha > 0$	$\frac{A}{\alpha + j\omega}$
$\begin{cases} 1 & -\alpha \le t \le \alpha \\ 0 & \text{otherwise} \end{cases}$	$\frac{2\sin\omega\alpha}{\omega}$
A constant	$2\pi A\delta(\omega)$
$u(t)A$	$A\left(\pi\delta(\omega) - \frac{j}{\omega}\right)$
$\delta(t)$	1
$\delta(t-a)$	$e^{-j\omega a}$
$\cos at$	$\pi(\delta(\omega + a) + \delta(\omega - a))$
$\sin at$	$\frac{\pi}{j}(\delta(\omega - a) - \delta(\omega + a))$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
$\frac{1}{t}$	$-j\pi\,\mathrm{sgn}(\omega)$
$\mathrm{e}^{-\alpha\|t\|},\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + \omega^2}$

Linearity:

$$\mathcal{F}\{f+g\} = \mathcal{F}\{f\}+\mathcal{F}\{g\}, \qquad \mathcal{F}\{kf\} = k\mathcal{F}\{f\}.$$

Shift theorems:

If $F(\omega)$ is the Fourier transform of $f(t)$

$$\mathcal{F}\{\mathrm{e}^{jat}f(t)\} = F(\omega - a), \qquad a \text{ constant.}$$

$$\mathcal{F}\{f(t-\alpha)\} = \mathrm{e}^{-j\alpha\omega}F(\omega), \qquad \alpha \text{ constant.}$$

Differentiation:

The Fourier transform of the nth derivative, $f^{(n)}(t)$, is $(j\omega)^n F(\omega)$.

Duality:

If $F(\omega)$ is the Fourier transform of $f(t)$ then

the Fourier transform of $F(t) = 2\pi \times f(-\omega)$.

Convolution and correlation:

The Fourier transform of $f(t) * g(t)$ is $F(\omega)G(\omega)$ where

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\lambda)g(t-\lambda)\,\mathrm{d}\lambda = g(t) * f(t).$$

The Fourier transform of $f(t) \star g(t)$ is $F(\omega)G(-\omega)$ where

$$f(t) \star g(t) = \int_{-\infty}^{\infty} f(\lambda)g(\lambda - t)\,\mathrm{d}\lambda.$$