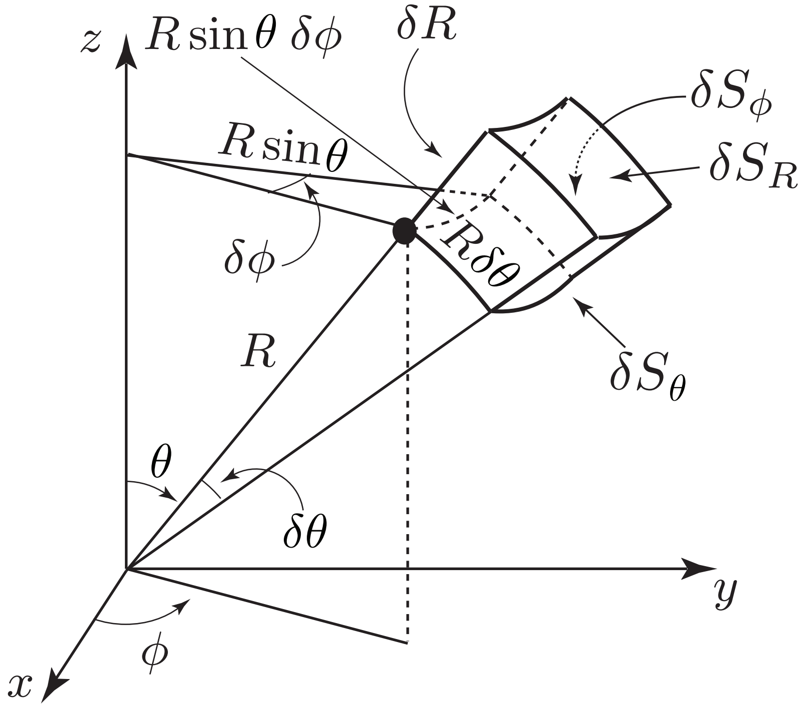
Spherical polar coordinates

The diagram shows spherical polar coordinates (R, θ, ϕ).



$$\left.\begin{array}{c} x = R\sin\theta\cos\phi \\ y = R\sin\theta\sin\phi \\ z = R\cos\theta \end{array}\right\} \qquad \begin{array}{c} R \geq 0 \\ 0 \leq \theta \leq \pi \\ 0 \leq \phi < 2\pi \end{array}$$

If $\mathbf{v} = v_R\hat{\mathbf{e}}_R + v_\theta\hat{\mathbf{e}}_\theta + v_\phi\hat{\mathbf{e}}_\phi$:

$$\nabla\Phi = \frac{\partial\Phi}{\partial R}\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\Phi}{\partial\theta}\hat{\mathbf{e}}_\theta + \frac{1}{R\sin\theta}\frac{\partial\Phi}{\partial\phi}\hat{\mathbf{e}}_\phi.$$

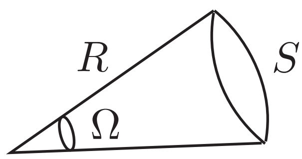
$$\nabla\cdot\mathbf{v} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 v_R\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(v_\theta\sin\theta\right) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\phi}\left(v_\phi\right).$$

$$\nabla\times\mathbf{v} = \frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{\mathbf{e}}_R & R\hat{\mathbf{e}}_\theta & R\sin\theta\,\hat{\mathbf{e}}_\phi \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ v_R & Rv_\theta & R\sin\theta v_\phi \end{vmatrix}.$$

$$\nabla^2\Phi = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{R^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}.$$

Volume element: $\delta V = R^2\sin\theta\,\delta R\,\delta\theta\,\delta\phi$.

Surface elements:
$\delta S_R = R^2\sin\theta\,\delta\theta\,\delta\phi$,
$\delta S_\theta = R\sin\theta\,\delta R\,\delta\phi$,
$\delta S_\phi = R\,\delta R\,\delta\theta$.



Solid angles:

Consider part of a sphere of radius R. If the area cut off on the surface is S, the **solid angle** at the centre is $\Omega = \frac{S}{R^2}$ steradians.

The solid angle at the apex of a cone of semi-vertical angle θ is $\Omega = 2\pi(1-\cos\theta)$.