Mathematical language

This introductory section provides useful background material on the importance of symbols in mathematical work. It describes conventions used by mathematicians, engineers, and scientists.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- understand how important it is to be precise about the symbols you use

- understand the importance of context when trying to understand mathematical symbols

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1. Introduction

Mathematics has its own language, much of which we are already familiar with. For example the digits

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

are part of our everyday lives. Whether we refer to 0 as 'zero', 'nothing', 'nought', or 'O' as in a telephone number, we understand its meaning.

There are many symbols in mathematics and most are used as a **precise** form of shorthand. We need to be confident when using these symbols, and to gain that confidence we need to understand their meaning. To understand their meaning there are two things to help us -

context - this is the context in which we are working, or the particular topics being studied,

and

convention - where mathematicians and scientists have decided that particular symbols will have particular meaning.

2. Some common mathematical symbols

Let us look at some symbols commonly associated with mathematical operations.

The symbol $+$.

Words associated with this symbol are 'plus', 'add', 'increase' and 'positive'.

As it stands, '$+$' clearly has some sort of meaning, but we really need to understand it within a context.

So, for example, if we see the $+$ symbol written in the sum

$$2 + 3$$

we understand that the context is one of adding the two numbers, 2 and 3, to give 5. So here, the symbol $+$ is an instruction to add two numbers together.

Let us look at another context in which we see the $+$ symbol.

If you study telephone numbers on business cards you will often see them given, for example, as

$$+44 \ 191 \ 123 \ 4567$$

In this context, the $+$ symbol means that, in addition to the usual telephone number, a person dialing that number from overseas will need to include the country code (in this case 44).

So we see that the $+$ symbol can have completely different meanings in different contexts, and it is important to be clear about the context.

The symbol $-$.

Words associated with this symbol are 'minus', 'subtract', 'take away', 'negative' and 'decrease'.

Again, to understand the symbol we need a context.

So, if we see the $-$ symbol written in the sum

$$6 - 4$$

we know this means 6 subtract 4, and we know the answer is 2.

In a different context, we might see $-5°$C, meaning a temperature of minus five degrees Celsius, that is five degrees below zero.

The symbol $\times$.

Words associated with this symbol are 'multiply', 'lots of', and 'times'.

This is really just a shorthand for adding. For example, if we see

$$6 + 6 + 6 + 6 + 6$$

we have five lots of six, or five sixes, and in our shorthand we can write this as 5×6.

Suppose we have

$$a + a + a + a + a$$

We might write this expression as $5 \times a$. However, in this context, especially in hand-written work, we may confuse the $\times$ symbol with the letter x, and so we would often write simply $5a$. We see that our shorthand has become even shorter. Multiplication is one of those rare occasions when we can omit a symbol altogether.

The division symbols

Division is symbolised in several different ways. For example

$$10 \div 5, \qquad \frac{10}{5}, \qquad 10/5$$

are three equivalent ways of writing ten divided by 5. We might also read this as 'how many times will 5 go into 10 ?'.

The $=$ sign and its variants

Another symbol used frequently is the equals sign $=$.

The $=$ sign does not mean anything on its own - we need a context.

For example, in the sum $1 + 2 = 3$, what we are saying is that whatever we have on the left-hand side is exactly equal to whatever we have on the right-hand side.

Variations on the equals sign are

$\neq$ which means 'is not equal to'

$\approx$ which means 'is approximately equal to'

$\geq$ which means 'is greater than or equal to',
e.g. $x \geq 2$ means that x can equal 2, but it might also be any value larger than 2.

$\leq$ which means 'is less than or equal to',
e.g. $y \leq 7$ means that y might equal 7 or might be any number less than 7.

3. Variables

Variables are another form of mathematical symbol. These are used when quantities take different values.

Imagine taking a car journey and think about the speed at which you are travelling. As you travel along your speed may change. So, speed is a **variable** - that is, a quantity which can change. We will be using letters to stand for quantities like this. For example, we might use the letter v for speed. To a large extent we can use any letter we choose, although there are conventions.

We might choose to use d for distance and t for time.

By convention, we use u to be an initial speed, and v to be a final speed.

In a different context, v might refer to volume. We need to check the context to fully understand the meaning.

If we see $v = \dfrac{d}{t}$ where $d =$ distance, and $t =$ time, then we would know that v is a speed.

On the other hand, if we see $v = \dfrac{4}{3}\pi r^3$ where r is the radius of a sphere, we know that v stands for the volume of the sphere.

Returning to our car journey. We might want to record the journey time on several different days. In this context we might choose to use a **subscript** and write

$$t_1, t_2, t_3, t_4, t_5$$

for the journey times on each of five different days. Alternatively we could write

$$t_m, t_t, t_w, t_h, t_f$$

for the journey times on Monday through to Friday. Note how we have used t_h for the journey time on Thursday to avoid confusion with t_t for the journey time on Tuesday.

So, a subscript is a small number, or other symbol, written to the bottom right of a variable to distinguish different instances of that variable.

4. The Greek alphabet

You will find that Greek letters are used in many calculations.

For example, the Greek letter 'pi', written π, is used to represent the number 3.14159.... This number continues forever without repeating.

We often use α ('alpha'), β ('beta'), and θ ('theta') to represent angles.

The Greek capital letter 'sigma' or Σ is frequently used to represent the addition of several numbers, and you will see it provided for this purpose on the toolbar of any spreadsheet program.

For future reference the full alphabet is given here:

The Greek alphabet

A	α	alpha	I	ι	iota	P	ρ	rho
B	β	beta	K	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	T	τ	tau
Δ	δ	delta	M	μ	mu	Υ	υ	upsilon
E	ϵ	epsilon	N	ν	nu	Φ	ϕ	phi
Z	ζ	zeta	Ξ	ξ	xi	X	χ	chi
H	η	eta	O	o	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

5. Some more symbols

The positioning of numbers and symbols in relation to each other also gives meaning. For example, you will come across use of **superscripts**. These are small numbers or symbols written at the top right of another, as in 4^2. In this context 4^2 is a shorthand for 'four squared' or 4×4. Similarly 4^3 is shorthand for 'four cubed' or $4 \times 4 \times 4$.

On the other hand $32°$ can mean different things in different contexts. It might mean an angle of 32 degrees. It could mean 32 to the power zero, which is actually 1. In printed work characters with slightly different shapes and sizes are used - compare $32°$ meaning 32 degrees and 32^0 meaning 32 to the power 0. Clearly these have very different meanings! You need to know the context! $32°$C is a temperature of 32 degrees Celsius.

What about 6,3 ? This could mean several things. But with brackets around, $(6, 3)$ can mean a pair of coordinates used to plot a point on a graph.

Brackets can mean a variety of different things. For example, in the study of probability you will come across expressions like $p(H) = \frac{1}{2}$ - this means the probability of scoring a Head, when tossing a coin, is $\frac{1}{2}$. Again, knowing the context is vital.

The symbol % is a percentage sign and means 'out of 100' as in 90% meaning 'ninety out of one hundred'.

$\sqrt{}$ is a square root sign. For example $\sqrt{16}$ is the number which when multiplied by itself is 16, that is 4 or -4.

$\overline{x}$, read as 'x bar' is the **mean** of a set of numbers.

$1.\dot{3}$ is a recurring decimal sign meaning that the 3's continue forever, as in 1.33333...

$1.\dot{3}1\dot{7}$ means that the 317 goes on forever, as in 1.317317317...

6. Summary

In summary, mathematical symbols are a precise form of shorthand. They have to have meaning for you. To help with understanding you have *context* and *convention*.