

Algebra

Removing brackets:

$$a(b + c) = ab + ac, \quad a(b - c) = ab - ac$$

$$(a + b)(c + d) = ac + ad + bc + bd \quad \left(\frac{a}{b}\right)c = \frac{ac}{b}$$

Formula for solving a quadratic equation:

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices:

$$a^m a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} \quad (a^m)^n = a^{mn} \quad (ab)^n = a^n b^n$$

$$a^0 = 1 \quad a^{-m} = \frac{1}{a^m} \quad a^{1/n} = \sqrt[n]{a} \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Logarithms: for any positive base b (with $b \neq 1$)

$$\log_b A = c \quad \text{means} \quad A = b^c.$$

Logarithms to base e, denoted \log_e or alternatively \ln are called *natural logarithms*. The letter e stands for the exponential constant which is approximately 2.718.

$$\log_e A \text{ or } \ln A = c \text{ means } A = e^c.$$

c , the natural logarithm of a number A , is the power to which e would have to be raised to equal A . Note:

$$e^{\ln A} = A \quad \text{if } A > 0; \quad \ln(e^A) = A$$

Logarithms to base 10: $\log_{10} A = c$ means $A = 10^c$.

pH: of a solution measures its acidity or basicity.

$$\text{pH} = -\log_{10}([H^+]/c^\ominus) \quad \text{so} \quad [H^+] = 10^{-\text{pH}} c^\ominus$$

where $[H^+]$ = hydrogen ion concentration in mol dm^{-3} and $c^\ominus = 1 \text{ mol dm}^{-3}$.

Equivalently, $\text{pH} = -\log_{10} a_{\text{H}_3\text{O}^+}$ where $a_{\text{H}_3\text{O}^+}$ = hydronium ion activity.

Laws of Logarithms: for any positive base b , with $b \neq 1$,

$$\log_b A + \log_b B = \log_b AB, \quad \log_b A - \log_b B = \log_b \frac{A}{B},$$

$$n \log_b A = \log_b A^n, \quad \log_b 1 = 0, \quad \log_b b = 1.$$

Formula for change of base: $\log_a x = \frac{\log_b x}{\log_b a}$. Specifically, $\log_{10} x = \frac{\ln x}{\ln 10}$.

Inequalities:

$a > b$ means a is greater than b

$a < b$ means a is less than b

$a \geq b$ means a is greater than or equal to b

$a \leq b$ means a is less than or equal to b

Sigma notation:

The Greek letter sigma, Σ , is used to abbreviate addition. If we have n values, x_1, x_2, \dots, x_n , and add them, the sum $x_1 + x_2 + \dots + x_n$ is written $\sum_{i=1}^n x_i$.

Note that i takes all whole number values from 1 to n .

So, for instance $\sum_{i=1}^3 x_i$ means $x_1 + x_2 + x_3$.

Example: $\sum_{i=1}^5 i^2$ means $1^2 + 2^2 + 3^2 + 4^2 + 5^2$.

Product notation: $\prod_{i=1}^n x_i = x_1 \cdot x_2 \cdot \dots \cdot x_n$.

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