Investigation of International Mathematical Cultures

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1. Background

Recruitment to post-graduate mathematics programmes and to lecturer positions in mathematics departments in UK universities has become dominated by international students and staff. In many UK university mathematics departments nine out of 10 appointments go to candidates from abroad [1]. The Pacific Rim and Eastern European countries are particularly prevalent, and there are significant numbers from other European countries. It was reported in 2005 that the shortage of mathematics teachers in UK schools was so bad that the Department for Education and Skills stopped collecting the figures [1]. Commenting upon the adequacy of the current three-year PhD model prevalent in the UK, the review [2] noted, 'The system of three-year PhDs can only work if there is excellent A level education at school level. Our perception is that A levels are weaker than they used to be. The result then is that this produces many students who cannot compete with graduates from abroad'. There is an increasing number of international (non-UK) students in UK HE. In 2011-12 they totalled 435,230 compared with 428,225 in 2010-11 (an increase of 2%), and made up 14% of full-time first degree students and 13% of all first degree students, 69% of full-time taught postgraduates and 46% of all taught postgraduates, 48% of full-time research degree students and 41% of all research postgraduates [3]. Among them, 19% did degrees in Mathematical Sciences [3].

Although mathematics is generally regarded as 'the universal language', the reality is that different countries have very different cultures when it comes to the teaching and learning of mathematics. There are significant variations in the pre-university mathematical experience, in terms of the curriculum content, learning styles, levels of abstraction and assessment methods [4]. Although the curriculum contents share some common ground, for example, all require the study of pure mathematics, the balance between breadth of application and pursuit of complex and deep study is vastly different [5]. For example, the current Hong Kong A Level Pure Mathematics covers a considerable portion of the analysis covered in first-year degree programmes in the UK. Of more significance than the content is the emphasis within the curriculum – in some countries, the focus is on procedural fluency, whilst in others it falls on conceptual understanding and/or on real-life applications and modelling. There are also more general cultural differences – for example, in UK examinations, multi-step questions are common whilst in China, students have to address whole questions, finding the sequence of logical steps by themselves (see examples in Appendices 2 and 3). In British textbooks, the same topic can appear repeatedly at different teaching stages, but the difficulty increases progressively. In China one topic normally appears only once and different topics are combined in logical and coherent order. In the UK, mathematics is only compulsory up to GCSE and currently only 15% of students in England continue mathematics afterwards, while in many other countries, students have to study mathematics throughout their school careers [6]. Likewise, in some cultures mathematics is regarded as the ultimate individual pursuit whilst in others group working is heavily promoted. Some systems typically offer a variety of different mathematics courses available to learners at senior secondary level, but there are no alternatives in some others. Even within the UK, a considerable number of pre-higher education mathematics qualifications are available and, for those working within HE sectors, it is not always clear what mathematics content, methods and processes students will have studied (or indeed can be expected to know and understand) as they commence their university level programmes [7]. With increasing numbers of international students and academic staff in UK HE, the scene is becoming more complicated. Students enter degree courses with a wide range of backgrounds and bring with them different experiences. At the same time, academic staff, having experienced different education systems, may have some unrealistic expectations from their students.

Consequently, the gap between students' actual mathematical knowledge and that perceived by their lecturers will, at the very least, impair the quality of their education and, at the worst, may prove too difficult for them to bridge [8]. This guide is to investigate the mathematical cultures of a range of the main international suppliers (of students and staff) of UK HE mathematics. The guide has not only focused on curriculum contents but has also addressed some often implicit issues of mathematical cultures, drawing upon personal experience of academic staff who have experienced more than two educational systems. The guide is of use to several different groups:

1. International students – the guide highlights to them the differences between the mathematics culture that they have been educated in and

that prevailing within UK universities. This helps them to prepare better for a UK degree course and thereby to achieve their full potential.

2. UK-educated staff – the outputs inform them of the mathematics cultural background of their students and enable them to make appropriate adaptations to their teaching and to educate their students about changes they may need to make to their approach to studying.

3. International staff – they benefit in the same way as both international students and as UK-educated staff.

2. Methodology and activities

We have aimed to draw together knowledge which is already held in diverse places (predominantly the personal experience of academic staff) within the UK HE mathematics community. The project leader, Dr Aiping Xu, was educated to master's degree level in China, studied for her PhD in France and now works as a mathematics support lecturer within **sigma**, the Centre for Excellence in University-wide Mathematics and Statistics Support at Coventry University. Coventry has a high proportion of overseas students, from over 100 different nations, and Dr Xu's work with students from all faculties brings her into contact with a wide range of nationalities and this first-hand experience has shown that certain aspects of mathematics are found to be extremely difficult by students from some countries and relative easy by those from other countries.

Phase 1 - Desk research

The initial phase of work on this project consisted of desk research gathering information about the pre-university mathematics education systems in a number of countries, particularly those countries from which many mathematics academic staff and post-graduates have been recruited in recent years. Eight colleagues from a range of countries, both European and international, agreed to participate in the preliminary interviews. A list of questions to guide semi-structured interviews was drawn and interviews were scheduled.

Phase 2 – Preliminary interviews

Eight pilot interviews with academic staff from a number of different countries (China, Italy, Germany, Ireland, Russia, Pakistan, and one from an English senior colleague who has rich experience in mathematics education) were undertaken. We have also received an email response from a Dutch colleague. We started with a list of questions based on the author's personal experience and adjusted it correspondingly along interviews. All the interviews were recorded to enable thorough post-interview analysis. The findings from these preliminary interviews were used to draw up a questionnaire for wider circulation to colleagues in mathematics departments throughout the country. A website for the delivery of the questionnaire was designed.

Phase 3 – Data gathering

The findings from the interviews were used to inform the design of a questionnaire to gather information from the wider community. The project website was established. This enabled the questionnaire (reproduced in Appendix 1) to be completed online from [9].

The online questionnaire included other appropriate features allowing international mathematics staff to contribute information about their experience of teaching and learning mathematics in the UK and back in their own countries. A project advertisement was designed and circulated amongst a wide community. A number of approaches were taken in order to ensure a good response. The advertisement was sent to all heads of mathematics departments and published in the MSOR newsletter. It also appeared on *Mathematics Today,* and the newsletter of the London Mathematical Society and its forum (the De Morgan forum). Moreover, personal emails were sent out to personal contacts to ensure a good response.

Phase 4 – Response analysis

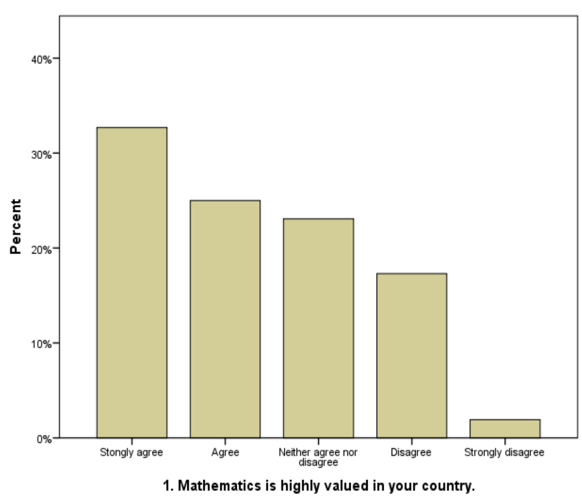
We received 52 responses from the online questionnaire. In some cases, we grouped 38 responses into four categories according to their social or educational characteristics as follows:

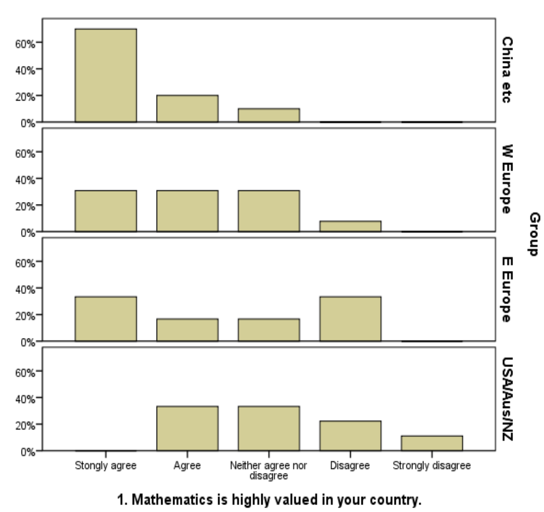
- China etc: 10 respondents including 8 from mainland China, 1 from Hong Kong and 1 from Singapore;
- West Europe: 13 respondents including 3 from Germany, 3 from Italy, 2 from France, 2 from the Netherlands, 1 from Portugal, 1 from Ireland and 1 from Cyprus;
- East Europe: 6 respondents including 3 from Russia, 1 from Hungary, 1 from Azerbaijan and 1 from Lithuania;
- USA/Aus/NZ: 9 respondents including 7 from USA, 1 from Australia and 1 from New Zealand

Some mathematics examination papers prior to higher education from some countries were collected and translated, which gave some additional insights into the mathematical cultures of those countries. We will now describe, question by question, our online questionnaire and analyse, overall and by groups, the responses that we have received so far.

1. Mathematics is highly valued in your country.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know



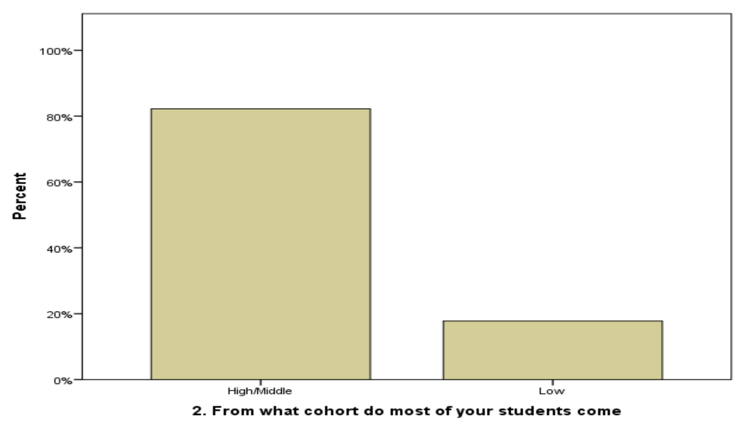


About 60.4% of the 48 people, who replied to this question, thought that mathematics was highly valued in their home countries while 16.7% did not think so. A formal comparison (Kruskal-Wallis test) revealed that there were statistically significant differences among some of the four groups.

Mathematics is commonly regarded as a mandatory subject if a student aims at a mathematics or science based degree. However, different countries have very different attitudes towards mathematics capability in general. In the UK, USA or New Zealand it is socially acceptable to say you cannot do mathematics. But in France, Italy, Russia or China, mathematics is highly valued at school from an early age. Pupils who are bad at mathematics are called losers and nobody wants to be a loser. Overall, mathematics is regarded as very demanding so it serves as the main subject to discriminate academically between students.

2. In terms of academic ability, most students who choose to study mathematics at university are from which part of the cohort?

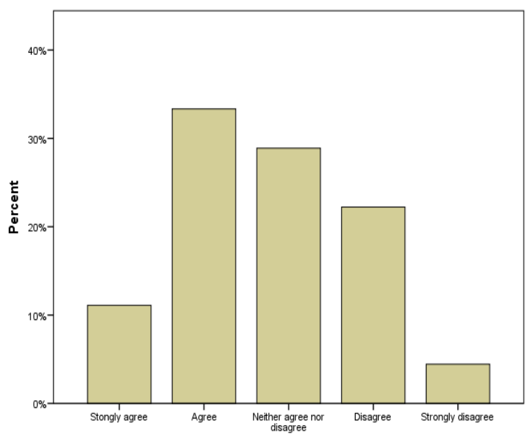
- ☐ High
- ☐ Middle
- ☐ Low
- ☐ Don't know



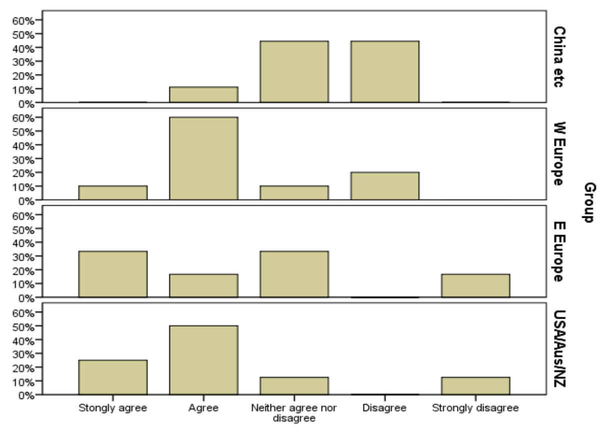
The majority of students studying mathematics at university come from high or middle cohorts in terms of academic ability. In some countries, since mathematics is perceived as a difficult subject, the attitude is, ‘don’t bother even trying, unless you are really smart’. Alternatively, schools discourage students from taking mathematics unless they are confident the students will do well because of ‘league table’ systems.

Some Australian universities have many overseas students since mathematics is seen as less language intensive although these students are not necessarily academically capable.

3. Academic staff in your country complain about the standard of incoming mathematics undergraduates.
- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know



3. Academic staff in your country complain about the standard of incoming maths undergraduates.



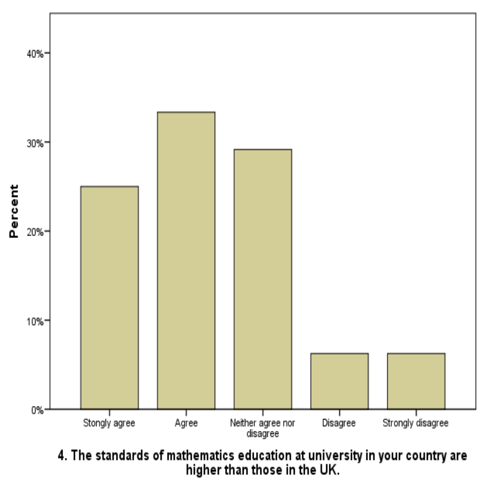
3. Academic staff in your country complain about the standard of incoming maths undergraduates.

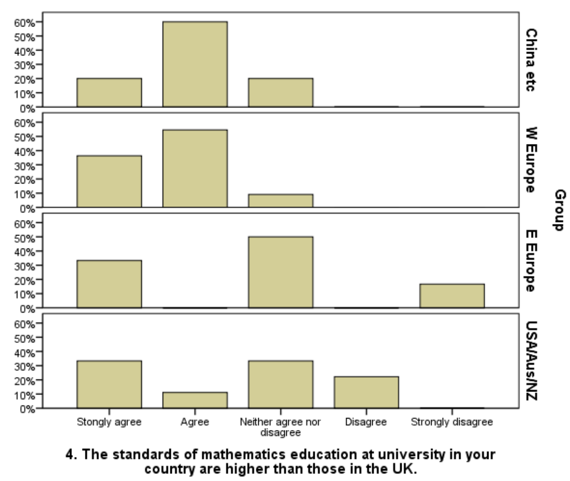
About 46.3% of the 41 people thought that academic staff in their home countries complained about the standard of incoming mathematics undergraduates while 24.4% disagreed. Academic staff assume that students today are taught the same as they were taught (probably it was many years ago) and they wish that their students were even better. Moreover, some governments' widening participation drives mean that universities often have to accept incoming students who are able but not educated well enough to be ready for university. However, the China etc group seems to have a different profile, which might be explained by the fact that complaining is not part of the culture and that the standard of incoming mathematics (or other disciplines) undergraduates is stable.

In England, research carried out by the Engineering Council [10] into the achievements of students taking A level mathematics shows that there is 'clear evidence' of a 'decline over time in the competency of students with the same A level grade'. The Council used a diagnostic test designed by Coventry University, consisting of 50 multiple-choice questions taken by 600 students per year. In 1991 those with a grade B at A level scored 40.5/50. Seven years later, in 1998, students scored just 36.8/50.

4. The standards of mathematics education at university in your country are higher than those in the UK.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know



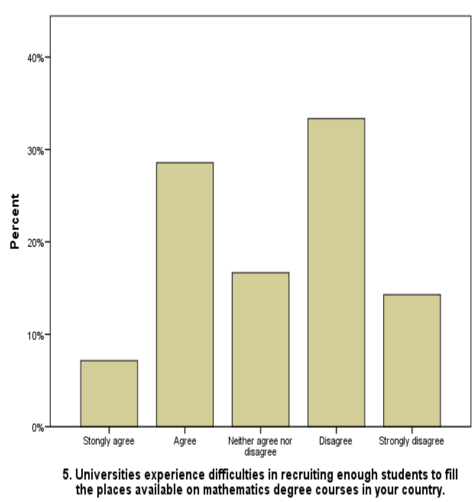


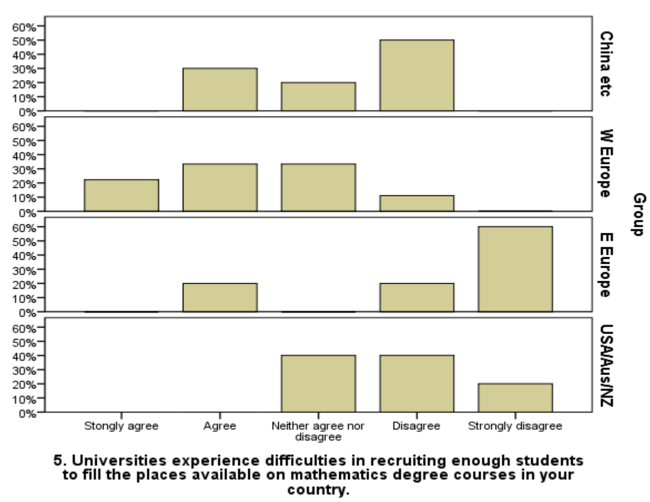
It is difficult to compare the standards of mathematics education at university, particularly when the range of universities is taken into consideration. However, 60% of the 45 people thought that the general standards of mathematics education at university in their home countries were higher than those in the UK while 13.4% did not. Kruskal-Wallis test showed that the differences between groups was statistically significant.

Many countries put more emphasis on pure mathematics compared with applied mathematics. In the US, students are required to complete regular written or computer-based assessment, which ensures that they are regularly engaging with the material throughout the semester rather than waiting until the revision period at the end of the year to begin revising. It is pointed out by a Russian colleague that UK students are about 3-4 years behind their rivals in Russia so a university degree in the UK is roughly equivalent to a standard school certificate in Russia. In the last International Mathematics Competition for university students [11], the top three teams were from Russia, Poland and Ukraine while Cambridge University took the 21st place.

5. Universities experience difficulties in recruiting enough students to fill the places available on mathematics degree courses in your country.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know



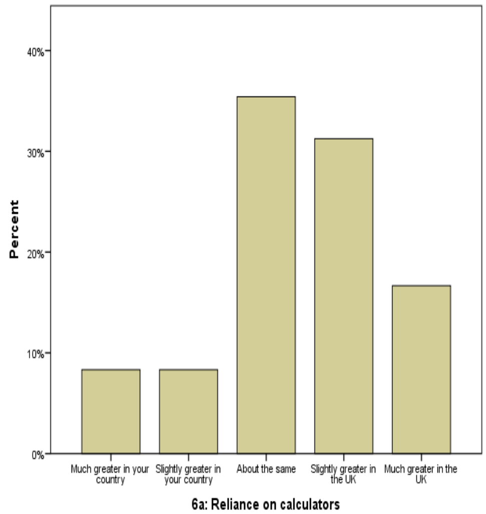


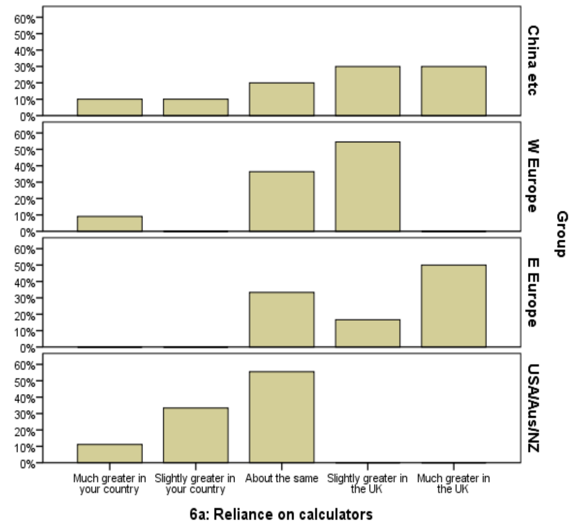
Among the 38 people who responded to this question, half of them did not think that universities experienced difficulties in recruiting enough students to fill the places available on mathematics degree courses in their home countries while 31.6% did, which are mainly in the West Europe group. Further testing showed that there was statistically significant differences among some of the groups. This clearly depends on the reputation of universities. However, further comments on this question suggest that, some countries have difficulties in student recruitment because a mathematics degree has the reputation to be hard and only useful for those with an interest in mathematics (but not as a general and good basis for all kinds of careers) and only few students take it. In some other countries, there is no shortage of students, because mathematics departments do a lot of service teaching and fill up their courses with students on related programmes, or simply there is a shortage of HE places as a whole.

6. A number of characteristics are listed below, in relation to each one please indicate whether there is a striking difference between incoming undergraduates to mathematics degrees in your country and those in the UK.

	Much greater in your country	Slightly greater in your country	About the same	Slightly greater in the UK	Much greater in the UK
Reliance on calculators	○	○	○	○	○
Experience of group working	○	○	○	○	○
Deep understanding	○	○	○	○	○
Mathematical common sense	○	○	○	○	○
Formal proof ability	○	○	○	○	○
Understanding of mathematical rigour	○	○	○	○	○

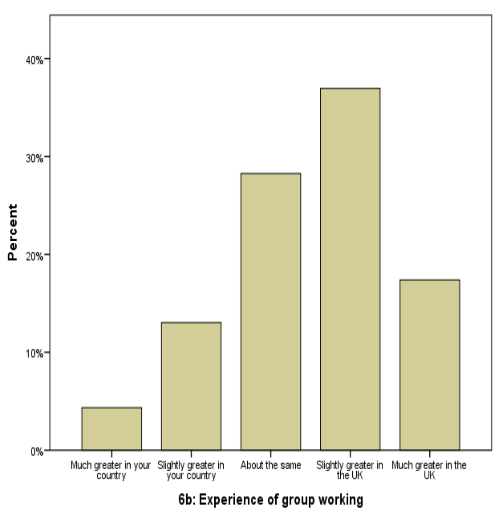
a. Reliance on calculators

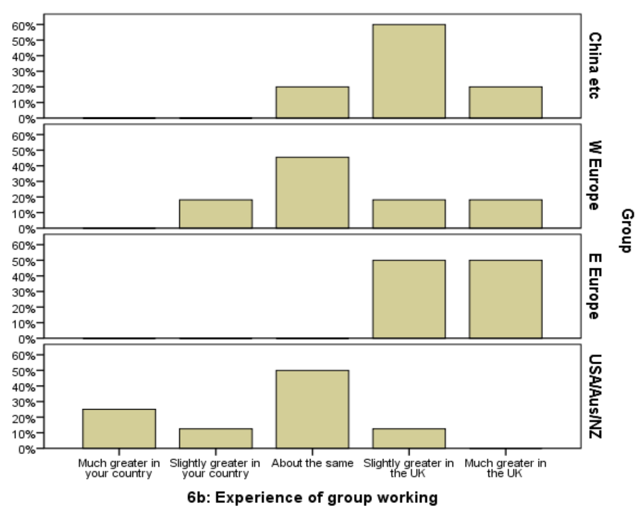




Among the 45 people who replied to this question, 48.9% of them thought that students' reliance on calculators in the UK was greater than their home countries while 18.8% disagreed, which were mainly from the USA/Aus/NZ group. Further testing showed that the difference was statistically significant. Kruskal-Wallis test revealed a statistically significant difference between some of the groups, which can be explained by the fact that in some countries such as the USA, calculators are allowed for major exams while in other countries, such as China, they are not.

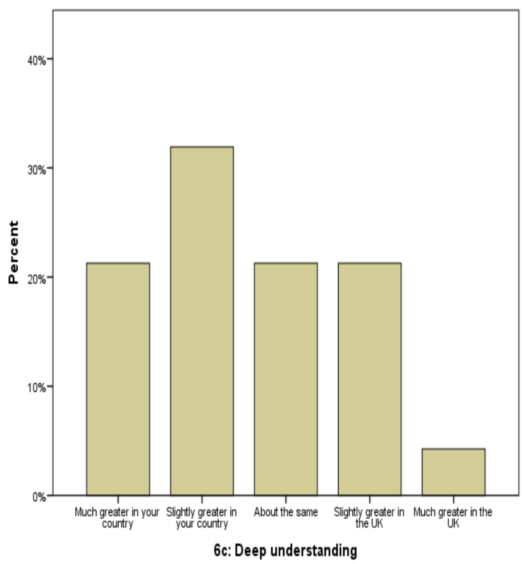
b. Experience of group working

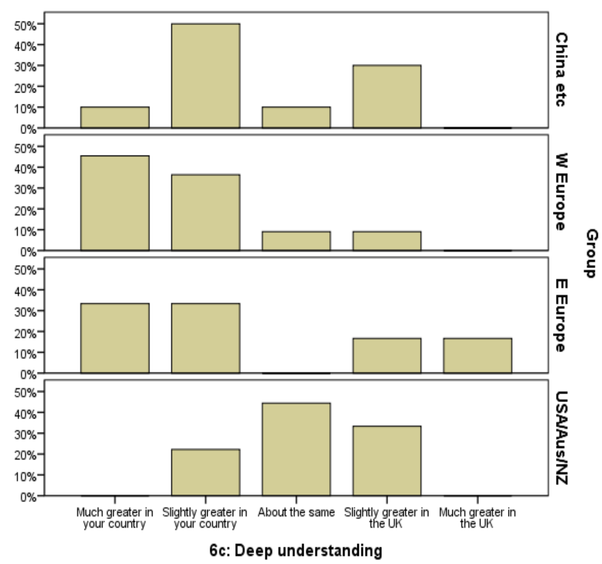




Among the 43 people who replied to this question, 55.8% of them thought that experience of group working was greater in the UK than in their home countries while 18.7% did not, which were mainly from the USA/Aus/NZ group. Kruskal-Wallis test revealed a statistically significant difference between some of the groups. In some cultures learning mathematics is regarded as an individual endeavour.

c. Deep understanding

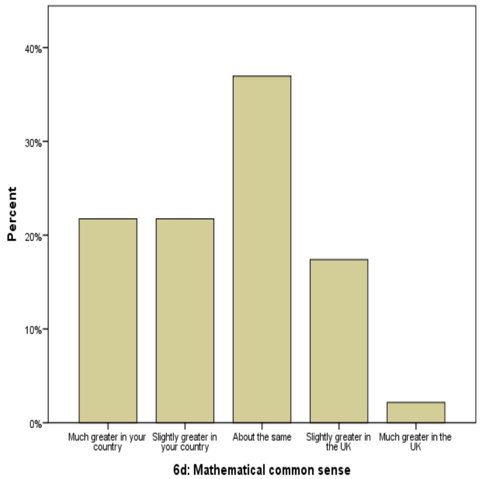


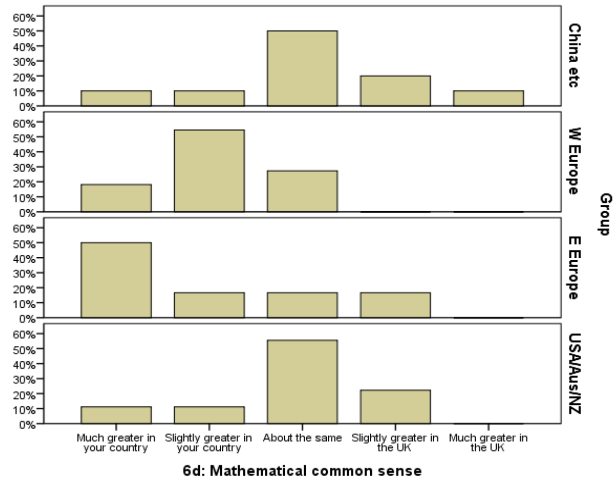


Among the 44 people who responded to this question, 56.8% of them thought that incoming undergraduates to mathematics degrees in their countries had deeper understanding than those in the UK while 27.2% did not. Kruskal-Wallis test revealed a statistically significant difference between some of the groups.

Some respondents says that there are some students who do not know basic mathematics concepts and that they are used to doing exercises without really understanding why. In some countries obsessive testing in schools has been damaging, as regard to deep understanding.

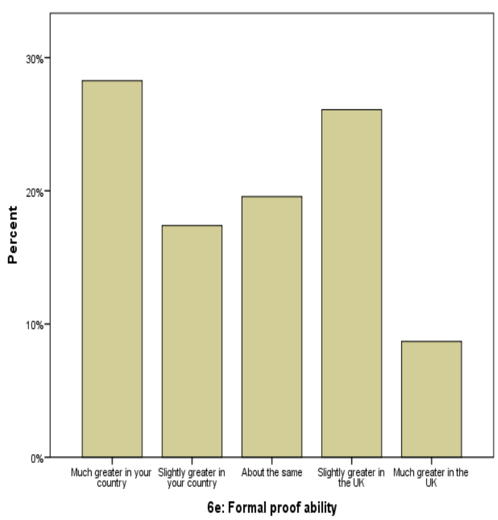
d. Mathematical common sense

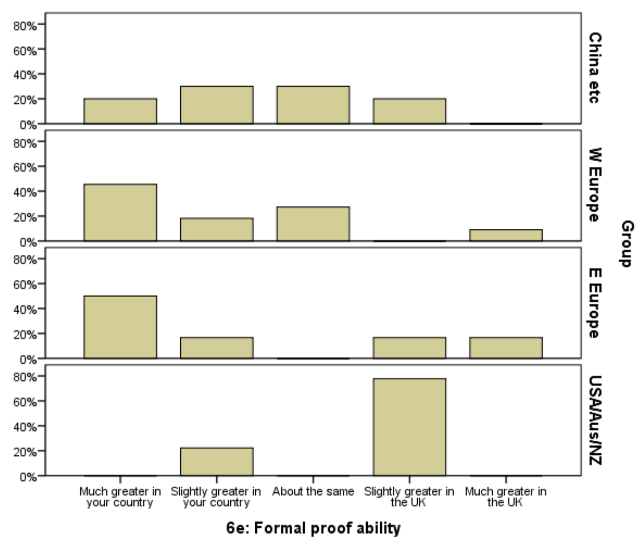




Among the 43 people who replied to this question, 44.2% of them thought that students in their home countries had more mathematical common sense than those in the UK while 20.9% did not. However, Kruskal-Wallis test showed that the difference between some of the groups was not statistically significant.

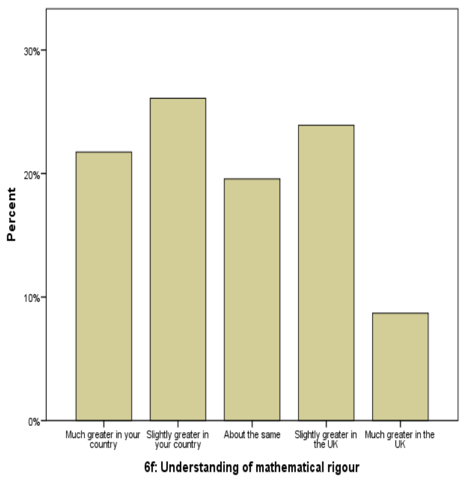
e. Formal proof ability

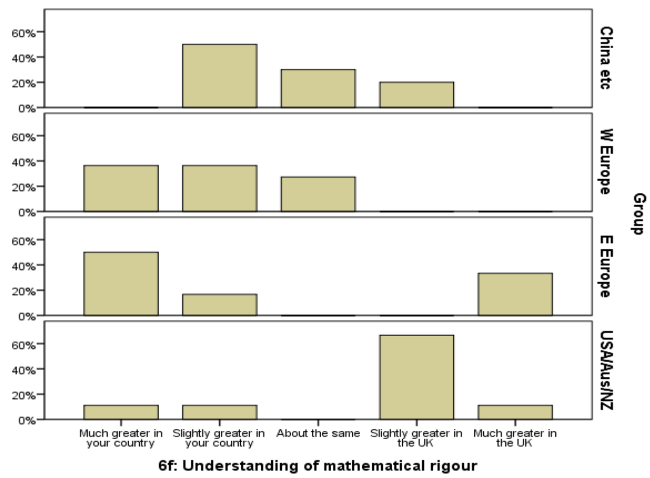




Among the 43 people who responded to this question, 48.8% of them thought that the formal proof ability of students in their countries were greater than those in the UK while 37.2% did not, which were mainly from the USA/Aus/NZ group. However, Kruskal-Wallis test showed that the difference was not statistically significant, which may be due to the fact that the USA/Aus/NZ group shares a similar model to the UK, which puts little emphasis on teaching formal proof. In China and some West European countries, school children are taught Euclidean Geometry and formal proof, which are regarded hard but useful to develop students' logical thinking and the benefits of which become more obvious as they progress in later years at university.

f. Understanding of mathematical rigour





More than half (51.2%) of the 43 people thought that students from their home countries had greater understanding of mathematical rigour than those from the UK while 34.9% did not. It is claimed that even the desire for rigorous argument seems to be missing in the UK. There is a statistically significant difference between some of the groups. No one from the West Europe group thought that UK students had greater understanding of mathematical rigour than their students. However, the majority (about 80%) of the USA/Aus/NZ thought that UK level was better.

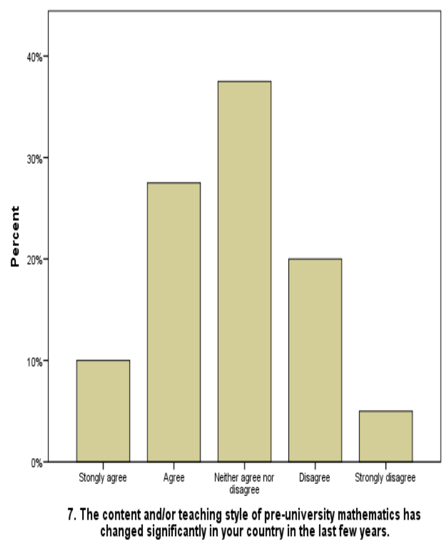
We also asked if there are other areas where there are striking differences and here are some of the responses:

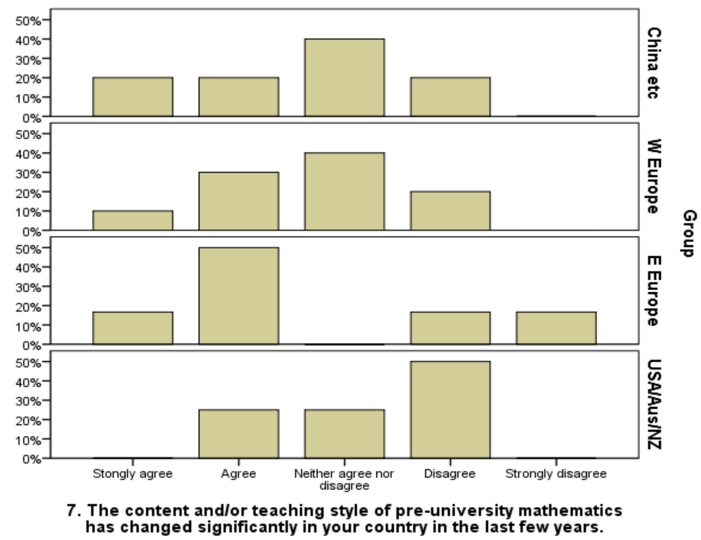
- 'In Russia, students have much better attention span, short-term and long-term memories, while the UK university students are comparable to the children with learning difficulties in auxiliary schools in Russia.'
- 'UK students expect to get enormous help at every step, from nursery to PhD. Russian students are more independent and they rely on their efforts and skills.'

- ‘The UK school system offers more specialisation from 16 onwards. In their last two school years, students in the UK study 3 or 4 A levels from a wide range of subjects and it follows that they can cover more material in each A level but they have narrower knowledge pool compared with some other countries such as in China students do 6 subjects (Chinese, Mathematics, English, Physics, Chemistry and Biology if they are science based) before entering university. ‘
- ‘US students are hungry for knowledge. Because they are paying, they demand to have things explained to them until they understand it. They ask lots of questions and demand to be taught. UK students are very polite, do not ask questions and they are paying for a finished product neatly delivered to their heads. ‘
- ‘This depends a lot on the institutions. For comparable institutions, in the US, new undergraduates are less critical of their university education and accept the fact that mathematics requires lots of hard work.’

7. The content and/or teaching style of pre-university mathematics has changed significantly in your country in the last few years.

- ○ Strongly agree
- ○ Agree
- ○ Neither agree nor disagree
- ○ Disagree
- ○ Strongly disagree
- ○ Don't know

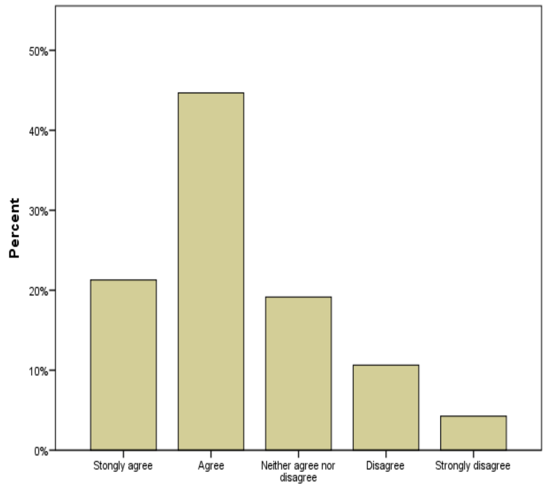




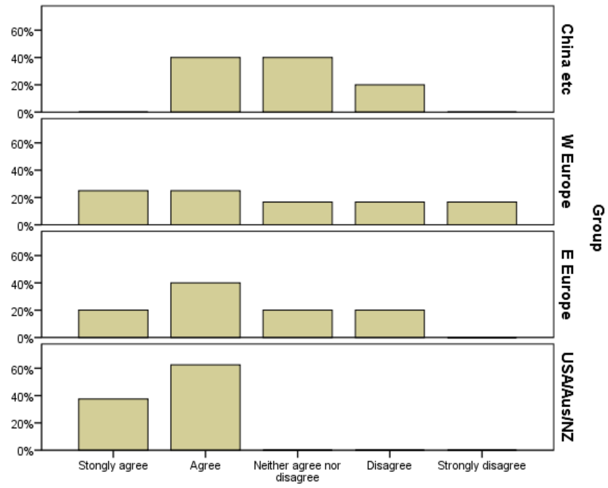
Among the 48 people who replied to this question, 12 people were not aware of recent changes back in their home countries since they had been away for a long time. About 38.9% of the people who responded agreed that the content and/or teaching style of pre-university mathematics had changed significantly in their countries in the last few years but 27.8% did not. Several respondents mentioned the detrimental effects of the introduction of unified tests across the country in Russia (following the American model). In many parts of Germany, the secondary school education has also been shortened by one year and consequently some topics have to be left out of the curriculum. In some countries, new topics have been added, such as probability and complex numbers.

8. Students with a mathematics degree have many career options open to them in your country.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know



8. Students with a mathematics degree have many career options open to them in your country.



8. Students with a mathematics degree have many career options open to them in your country.

In general, career options for students with a mathematics degree are really positive. 62.8% of the people thought that students with a mathematics degree had many career options open to them in their countries and only 16.3% did not. In the US, all responses were positive and it was said that mathematician was just listed as the number one job. Mathematicians are hired for non-mathematical skills, such as clear, rigorous and logical thinking, in many sectors such as IT, Finance, Insurance, Banking and other jobs heavily dependent on computers.

9. What things (practices, approaches, ethos) would you import from your country to here if you could?

There are some issues raised in different aspects of mathematics education.

- Formal proof and more mathematical rigour
  Several people have shown their concerns of UK students lacking the basic capability of formal proof. Students are not used to doing any abstract mathematics. Schools and universities should place more emphasis on theoretical approaches.
- Mathematics teachers' training
  Mathematics teachers should be better trained with higher levels of STEM knowledge and achievement starting from primary school teachers. In China many primary schools have mathematics specialist teachers.

- Lengthen degrees
  'Many undergraduates lack mathematical writing and critical thinking skills, and currently there is little room to develop these in the mathematics course.' Some respondent suggested introducing longer Master's degrees which can be broader at the beginning with taught components and then have several options for specialisation later. In China, it normally takes four years for a bachelor's degree and three years for a Master's.
- Textbooks
  'In the UK, it is more difficult to require use of a textbook, which means in many cases lecturers have to spend unnecessary time writing lecture notes for a course for which there already exists a very good textbook, time which could otherwise be spent engaging directly with students. A good lecture should really be more like a tutorial, in which students come prepared: having read some portion of a textbook and attempted some exercises on their own first.'
- Continuous assessment
  'Continuous assessment should be strengthened in the form of, say, weekly assessed homework/assignments. Allow qualified undergraduates to assist with the extra marking duties and this is good practice for them as well. Place more weight on continuous assessment rather than final degree exams. In honours courses in the US, maybe 40% of the assessment would come from a final degree exam. The rest would come from mid-term exams and/or homework/assignments. This ensures that students are engaging in the course early and often throughout the semester. Active, continual engagement is particularly crucial in a subject such as mathematics, which is so cumulative in nature.'
- More freedom for teachers to teach
  'Results related impositions force teachers to deliver only the material the bottom set can swallow, leaving the rest unattended.'
- 'Removal of students' questionnaires, especially NSS (the National Student Survey). The main job for lecturers is to teach, not to make students happy.'
- 'Oral test and more mathematical thinking based work.'
  These will force students and even staff to memorise the material and develop a habit of defending one's ideas. '
- 'Harsher grades'

- ‘Surprising hand-ins during tutorials’
- More practice and more contact hours
  ‘Modules should be accompanied by seminars. It is a good way to help weak students manage their time and make progress early on in their degree programme.’

3. Detailed study of some examinations

In this section, we will have a close look at the major examinations that students take before they enter universities in some representative countries, because what is assessed in examinations and qualifications, and the assessment techniques used to test students, are critical parts of any education system [5].

- **England**

Issues regarding the teaching and learning of mathematics remain, and the United Kingdom is still one of the few advanced nations where it is socially acceptable – fashionable, even – to profess an inability to cope with the subject.

- Sir Peter Williams [12]

Since their introduction in 1951, the Advanced Level General Certificates of Education, popularly known as A levels, have functioned as England’s principal pre-university examination. A levels are available in over 45 subjects. On average students take three or four A levels at around age 18 [5].

A Level qualifications are examined by three awarding bodies in England: Assessment and Qualifications Alliance (AQA), Edexcel and Oxford, Cambridge and RSA (OCR). The large majority of students entering HE have taken GCSE and A Level qualifications, but several other qualifications are also used as routes into HE. A level mathematics consists of 6 modules of equal size. Students can resit individual modules to improve their marks. All modules are available in June with some also available in January [7]. However, this procedure is under review.

According to the Nuffield Foundation [6], the UK has the smallest proportion of 16-to 18-year-olds studying mathematics of any of the 24 countries examined: far less than nations such as France, the US, Ireland, New Zealand, Russia, Australia, Estonia, Spain, Germany or China. 85% of Japanese students are studying the equivalent of A Level mathematics – in England it is just 12% of young people.

In [13], a typical A level mathematics exam problem was given together with an example from a textbook which mimics the problem (reproduced in Appendix 2), which criticised textbooks specially tailored for particular examination boards. A level mathematics has also been criticised by many educators for being so bite-sized that pupils no longer make links between different topics that make the entire subject come together and make sense. Many students cannot see mathematics as a 'build on' subject. Even the brightest 18-year-olds at top universities are struggling. Academics warn that too many students are arriving to study mathematics or mathematics-related degrees without the basic mastery that they need – which inevitably means that they struggle with the demands of a university course. Instead of strengthening what is taught up to age 16, A level was simplified by being chopped into modules. This gives no time for ideas to settle, conceals the crucial connections between different topics, and reduces the possibility of using simple techniques from different modules to solve interesting problems [14].

However, reforms have recently been proposed to address the problems with a linear A level [15]. The move will address concerns about pupils sitting exams in modules, and re-sits leading to grade inflation. However, some people are quite happy with the A levels as they are now. Geoff Lucas, secretary of the Headmasters' and Headmistresses' Conference, which represents the heads of top public schools, said that although many of his members favoured a linear approach, modular A levels remained "extremely popular" with parents and pupils. They felt they were the best way to maximise grades and some schools would continue with them.

- **China**

Of all international comparability work, it is the international achievement tests which have most captured the attention of governments, policy-makers and media outlets. These include the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). East-Asian countries continue to lead the world in mathematics achievement. With China's debut (Shanghai) in international standardised testing the PISA 2009 tests showed that Shanghai was top of the international education rankings.

In China, education is traditionally highly valued. People embrace education as a real asset. Parents have high aspirations for their children and adhere to an ingrained view that hard work and putting up with hardship is the route to success [16]. This relates to the common Chinese belief that 'diligence can compensate for stupidity', or in other words, that it is effort and hard work that determine success not innate ability [16] [17].

In this cultural context, China has one of the most centralized curriculum systems in the world. Not only is there a national curriculum laid down, but until the late 1980s all students in China used the same set of textbooks. The curriculum in China focuses on building strong foundational knowledge and mastery of core concepts. All teaching and learning activities are clearly aligned to the national standards.

Teachers in China are highly respected by everyone, especially by students and their parents. A Chinese class typically has around 40-50 students with all of the desks facing the teacher and it is regarded as very important that teachers should see all their students' eyes to ensure all students get the message. The vast majority of teachers put content knowledge ahead of pedagogical knowledge, even though they know that the latter is also important. Student discipline has never been an issue. People believe in the importance of learning. Apart from doing their daily homework, Chinese pupils join out-of-school classes. They have a real attitude that 'education can change my life.'

Many elementary school students learn mathematics from a specialized mathematics teacher. This is not universal but common. Making connections between content areas is regarded as crucial in the teaching and learning of mathematics. The use of multiple methods to solve a mathematical problem is widely applied, as is the practice of giving classroom exercises, and examination and test questions in a variety of formats and structures. Teaching with variation has been applied either consciously or intuitively in China for a long time [18].

The education system in China is a 6-3-3(6 years for primary school, 3 years for junior secondary school and 3 years for senior secondary school) or 5-4-3 system (5 years for primary school, 4 years for junior secondary school and 3 years for senior secondary school). Years 1 to 9 education are compulsory. National and provincial testing and examinations have great Influence on and, in some cases, determine what students study and what competencies they are expected to achieve. In [19], a detailed description was given on the teaching and learning of school mathematics in China before 2000 and most importantly some sample papers for major local and national examinations were also provided. There are two major exams for Chinese students to take at the end of junior secondary school (Zhongkao) and senior secondary school (Gāokǎo) that dictate entrance into senior secondary schools and universities, respectively. In addition, at the local district or city level there is a universal final exam given at the end of each academic year that is voluntary, though most schools participate.

The National University Entrance Examination, Gāokǎo, is the standard means of entry to Chinese universities. It takes place annually across the country simultaneously over a three-day period in June. Mathematics is one of the three compulsory subjects although there are two different papers depending on the applicants' choices of sciences/engineering or art/humanities course. The examination is a terminal paper after three years of senior secondary education and calculators are not allowed. There is a significant amount of 3D geometry focusing on geometrical reasoning, which is regarded as highly demanding. Here is an example taken from the BBC website [20]:

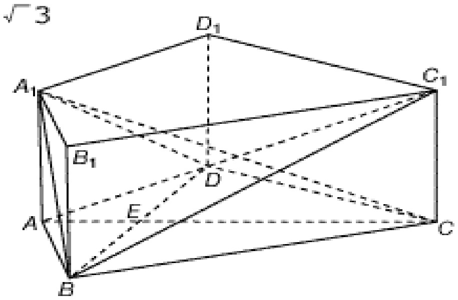
National test set by Chinese education authorities for pre-entry students

As shown in the figure, in square prism $ABCD—A_1B_1C_1D_1$, $AB=AD=2$, $DC=2\sqrt{3}$, $AA_1=\sqrt{3}$ $AD\perp DC$, $AC\perp BD$, and foot of perpendicular is E,

(i) Prove: $BD\perp A_1C$:

(ii) Determine the angle between the two planes A_1BD and BC_1D:

(iii) Determine the angle formed by lines AD and BC_1 which are in different planes.



Like many countries where opportunities for HE are limited, China's school system is heavily focused on selecting the most talented students to go on to the next stage. Given the fact that the intensity of the competition for limited university places is unimaginable, most senior secondary schools are evaluated by their academic performance in Gāokǎo by parents and students. Gāokǎo is regarded as gruelling and viewed as the gateway to adult success - there is a lot of competition and students had better be well prepared.

The university entrance examination system sometimes diminishes the importance of subjects that are not tested. Critics also say that the examination-driven system places undue emphasis on speed and memorization of obscure facts but does not produce the kind of students who are able to apply their knowledge in rapidly changing situations in a modern economy. In the 1990s, and especially after 2000, the system was significantly changed and some of the central control relaxed. In 2005, authorities of fourteen provinces, municipalities and autonomous organized their own separate university entrance examinations.

Traditionally, the Chinese senior secondary school (Years 10–12) curriculum in mathematics, building on the elementary and junior secondary curriculum, consisted of two distinct, mandatory series, each consisting of several courses: one in algebra and the other in geometry. This curriculum has been reformed to remove some of the most difficult topics and allow for some choice. Some new content has been added (for example, algorithms, some topics of statistics and probability, some topics of calculus, some topics taking as optional, etc.). Meanwhile, complexity of formula and terminologies, difficulty level and speed [21], were decreased. The algebra and functions topics have a very analytical focus and there is an emphasis on formal proof [5].

Among 65 countries, Shanghai 15-year-old students ranked top in mathematics in the 2009 administration of PISA. This was the first and only time that China has participated in international standardised testing. The results from an analysis of the Shanghai Zhongkao (senior secondary school entry exam) mathematics paper might give some insight into Shanghai students' good performance in PISA mathematics. Some sample questions in Shanghai Zhongkao 2011 can be found in [22]. Shanghai administers its own Gāokăo examination and a translated past paper in mathematics is enclosed in Appendix 3.

There are many universities in Australia that accept the Gāokăo score for applications. It is said that more and more Chinese students are taking the SATs (American exam) to skip the Gāokăo. Many high-performing secondary schools in bigger cities now have special classes for students who have decided to apply for American universities. Some students, who have prepared both exams, say that the SATs are regarded as child's play compared to the Gāokăo.

Some Chinese educators have criticised the Chinese education system for setting too great an emphasis on testing and for producing students who lack curiosity and the ability to think critically or independently.

- **France**

The Baccalauréat Général (or le bac) is an academic qualification which French and international students take at the end of the secondary education typically at the age of 18. It imparts a rounded secondary education, gives access to a wide range of university degree courses and differs from British A levels that cannot be obtained in single subjects [23].

There are three main orientations: S (Sciences), ES (Economics and social sciences) and L (Literature, languages, philosophy and arts). Three years are needed for the upper secondary school. Differentiations among orientations start at the second year and are reinforced in the third (last) year with the choice of 'Specialty' (Mathematics, Physics or Biology for the scientific orientation). Each orientation has an element of mathematics but the level of difficulty and subject weighting (which counts to the final certificate) varies. The mathematics in the sciences stream is the most demanding of the three. There is a national syllabus for each year and orientation. There were some important curricular changes in the 2000 High School Reform, including an increased emphasis given to statistics and probability, differentiation etc. A recent le bac mathematics paper is translated and enclosed in Appendix 4.

In [24], some interesting comparisons were made of the English and French didactic traditions in teaching mathematics. In England, pupils are often encouraged to use the shortest and simplest method for relatively straightforward problems as a way of easing them into the work. This emphasis on pragmatic methods fails to provide the rigour and foundations on which to build mathematical ideas and it denies some access to the full curriculum. However, French teachers take these opportunities to induct learners into the theoretical context and the full complexity of mathematics.

- **USA**

Most US universities require students to take college readiness tests, for example the SAT (Scholastic Assessment Test) and the ACT (American College Testing) in support of their applications. Traditionally, the SAT has been the most accepted test, although the ACT has seen an increased popularity. The ACT is more academically inclined. Students have the option to take either test (or both), and take them multiple times. The ACT is an achievement test, measuring what a student has learned in school. The SAT is more of an aptitude test, testing reasoning and verbal abilities. The ACT has up to 5 components: English, Mathematics, Reading, Science, and an optional Writing Test. The SAT has only 3 components: Critical Reasoning, Mathematics, and a required Writing Test. The College Board introduced a new version of the SAT in 2005, with a mandatory writing test. ACT continues to offer its well-established test, plus an optional writing test. The SAT has a correction for guessing. That is, they take off for wrong answers. The ACT is scored on the number of correct answers with no penalty for guessing [25].

Each SAT Subject Test consists of 50 multiple-choice questions with five choices. The mathematics questions are designed to test the mathematical competence of students who have studied college-preparatory mathematics—Level I for three years of study and Level II for more than three years of study. Both levels of the Mathematics Subject Tests measure understanding of elementary algebra, three-dimensional geometry, coordinate geometry, statistics, and basic trigonometry. The Level II exam tests these topics with more advanced content. In addition, the emphasis on various topics varies between the two levels. Half of the questions at Level I are directed at algebra and plane geometry and another quarter of the questions measure understanding of coordinate geometry and functions. At Level II, on the other hand, plane geometry is not tested at all, but nearly half of the questions are concentrated on trigonometry and functions. Level II devotes twice as many questions to miscellaneous topics as Level I [26]. Many practice papers for both levels can also be found there.

The ACT Mathematics Test is a 60-question, 60-minute test that requires students to use reasoning skills to solve practical problems in mathematics. The test assesses students' knowledge of high school level Algebra, Geometry, Co-ordinate geometry and Trigonometry. The questions are also of multiple-choice type and there is no negative marking for a wrong answer. Students are allowed to use calculator. The Mathematics questions of ACT are usually more difficult than other similar examinations [27]. Some ACT sample paper can be found at [28].

- **Russia**

Traditionally, the universities and institutes in Russia conducted their own admission tests regardless of the applicants' school record. There was no uniform measure of graduates' abilities; marks issued by high schools were perceived as incompatible due to grading variances between schools and regions. In 2003, the Russian Ministry of Education launched the Unified State Examination (USE) programme. This set of standardised tests for high school graduates, issued uniformly throughout the country and rated independently of the student's schoolmasters, akin to North American SAT, was supposed to replace entrance exams to state universities. Thus, the reformers reasoned, the USE empowers talented graduates from remote locations to compete for admissions to the universities of their choice, at the same time eliminating admission-related bribery, then estimated at 1 billion US dollars annually [29].

A student can take USE in the Russian language, mathematics, foreign languages (English, German, French, and Spanish), physics, chemistry, biology, geography, literature, history, the basics of social sciences and computing science. USE in the Russian language and mathematics are obligatory; that means that every student needs to get the necessary results in these subjects to enter any Russian university [30].

The traditional entry exams to some universities were highly selective and forced students to make additional efforts in order to improve their chances to successful admission. Since 2009, all Russian universities are obliged to admit students on the basis of results of the USE, the national standardized tests on different subjects. There is a lot of literature on it [31].

The USE is criticized as a poor measure of academic aptitude, which is having a detrimental effect on learning in schools because of the test preparations for the last whole school year. Like all standardized tests, it is regarded as a very simplistic way of testing knowledge, and it does not reflect the completeness and the range of knowledge and how they should be tested [32].

4. Discussion

Like music, success in mathematics depends on systematic, cumulative learning, and each new skill needs to be built on a solid foundation laid at earlier stages [13].

‘Mathematics is a Tall Subject’

William P. Thurston [33]

‘The structure is not like a tree, but more like a scaffolding, with many interconnected supports. Once the scaffolding is solidly in place, it is not hard to build it higher, but it is impossible to build a layer before previous layers are in place. Difficulties arise because students taking a particular course are in different stages of mastery of the earlier learning.’

‘Mathematics is a Broad Subject’

William P. Thurston [33]

‘The acceleration of the curriculum has had its cost: there has been an accompanying trend to prune away side topics. It used to be standard to study solid geometry and spherical geometry along with plane geometry. These topics have long been abandoned. The shape of the mathematics education of a typical student is tall and spindly. It reaches a certain height above which its base can support no more growth, and there it halts or fails.’

The Education Minister Elizabeth Truss gave a speech at the North of England Education Conference on 17 January 2013 [34]. She outlined the importance of mathematics education and she mentioned that all the evidence from international tests and league tables suggested that high performing countries put core academic subjects at the centre of their curriculum for longer than we do in this country. Consequently, UK students are less competitive in some areas than their international peers. Moreover, we are encountering a complicated situation with educational reforms across the world and with many international academic staff and students working and studying in the UK. Academic staff have some expectations about their students' mathematics knowledge, which sometimes is not valid owing to their different pre-university background and working experience. On the other hand, some students come to UK universities unaware that the mathematical culture is different from what they have experienced at school. We have drawn on international academic staff's personal experience in the hope of giving a more accurate picture. Moreover, we have provided some insights into how mathematics is taught and assessed in some other countries, which will help us know our students better and tailor our teaching accordingly. However, we acknowledge that some opinions of the respondents can be biased, and so more evidence-based research needs to be carried out in the future.

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Appendices

Appendix 1: Online questionnaire

Investigation of international mathematical cultures

A growing number of international students study mathematics at UK universities. Although mathematics itself may be the same the world over, the subject is learnt within a cultural setting and different countries have different mathematical cultures. The purpose of this project is to try to identify key ways in which the mathematical culture of other countries differs from that in the UK, so that both academic staff and students can be made more aware of these differences and so that appropriate induction can be provided. If you have experience of mathematics education in more than one country apart from the UK please complete a separate questionnaire for each country. Please note that completion of this questionnaire is completely voluntary. By completing the questionnaire you are indicating that you agree that anonymised quotes can be used as part of the research project. All the information that you provide will be treated in confidence. If you would like further information about the study, please contact Dr Aiping Xu (telephone 024 7688 7590 or email aiping.xu@coventry.ac.uk). In a number of items in this questionnaire you will be presented with a statement and then six options about your level of agreement with the statement - please select the one that most closely represents your opinion.

---

*Required

Please indicate which country you are going to provide information about. * If you have experience of mathematics education in more than one country apart from the UK please complete a separate questionnaire for each country. ☐

What is your mathematical experience from this country? [Select as many as apply] *

- ☐ School education
- ☐ University education (1st degree)
- ☐ University education (masters or doctorate)
- ☐ Post-doctoral research
- ☐ Lecturing experience

1. Mathematics is highly valued in your country.

- ○ Strongly agree
- ○ Agree
- ○ Neither agree nor disagree
- ○ Disagree
- ○ Strongly disagree
- ○ Don't know

2. In terms of academic ability, most students who choose to study mathematics at university are from which part of the cohort?

- High
- Middle
- Low
- Don't know

3. Academic staff in your country complain about the standard of incoming mathematics undergraduates.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know

4. The standards of mathematics education at university in your country are higher than those in the UK.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know

5. Universities experience difficulties in recruiting enough students to fill the places available on mathematics degree courses in your country.

- Strongly agree
- Agree
- Neither agree nor disagree
- Disagree
- Strongly disagree
- Don't know

6. A number of characteristics are listed below, in relation to each one please indicate whether there is a striking difference between incoming undergraduates to mathematics degrees in your country and those in the UK.

	Much greater in your country	Slightly greater in your country	About the same	Slightly greater in the UK	Much greater in the UK

	Much greater in your country	Slightly greater in your country	About the same	Slightly greater in the UK	Much greater in the UK
Reliance on calculators	○	○	○	○	○
Experience of group working	○	○	○	○	○
Deep understanding	○	○	○	○	○
Mathematical common sense	○	○	○	○	○
Formal proof ability	○	○	○	○	○
Understanding of mathematical rigour	○	○	○	○	○

7. The content and/or teaching style of pre-university mathematics has changed significantly in your country in the last few years.

- ○ Strongly agree
- ○ Agree
- ○ Neither agree nor disagree
- ○ Disagree
- ○ Strongly disagree
- ○ Don't know

8. Students with a mathematics degree have many career options open to them in your country.

- ○ Strongly agree
- ○ Agree
- ○ Neither agree nor disagree
- ○ Disagree
- ○ Strongly disagree
- ○ Don't know

9. What things (practices, approaches, ethos) would you import from your country to here if you could?

10. Please list all the countries in which you have first-hand experience of mathematics at some stage of the education system. If you have experience of more than one country (apart from the UK) please complete a questionnaire for each country but only answer questions 10-12 once.

11. Of the countries you listed in Q10, where would you say that mathematics standards are the highest?

12. Of the countries you listed in Q10, where would you say that mathematics is most valued in society as a whole?

We may wish to follow up some of the information given in this questionnaire by contacting you directly. If you would be willing to be contacted please give your email address below. If you leave this box blank then we will not contact you. Thank you.

Appendix 2: A level Mathematics question

A problem from Paper 6671 Edexcel GCE, Pure Mathematics P1 A/AS, 4.11.03:6

6. A container made from thin metal is in the shape of a right circular cylinder with height *h* cm and base radius *r* cm. The container has no lid. When full of water, the container holds 500 cm^3 of water.
(a) Show that the exterior surface area, *A* cm^2, of the container is given by

$$A = \pi r^2 + \frac{1000}{r}.$$

(4)
(b) Find the value of *r* for which *A* is a minimum. (4)
(c) Prove that this value of *r* gives a minimum value of *A*. (2)
(d) Calculate the minimum value of *A*, giving your answer to the nearest integer. (2)

A problem from a well-known and widely available textbook: Edexcel

Edexcel

12 The diagram shows a prism whose cross-section is a right-angled triangle with sides $3x$, $4x$ and $5x$ cm. The height is h cm.

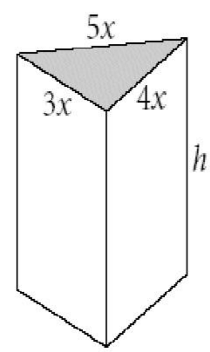
(a) **(i)** Show that the volume, V cm^3, is given by $V = 6x^2h$.

(ii) Show that the surface area, S cm^2, is given by $S = 12x^2 + 12xh$.

(b) Given the the volume of the prism is 100 cm^3, show that

$$S = 12x^2 + \frac{200}{x}$$

(c) Find, correct to three significant figures, the value of x for which S is a minimum, showing that this value does give a minimum for S.



Appendix 3: National University Entrance Examination (Shanghai)

2012 National University Entrance Examination

Shanghai, Mathematics (Engineering, agriculture and medicine)

Duration of the test: 2 hours

23 questions and 150 marks

No calculator is allowed

1. Fill in the blanks (14 questions and 56 marks)

1) Calculate $\frac{3-i}{1+i}$ =___(*i* is the imaginary unit).

2) If the set $A=\{x \mid 2x+1>0\}$， $B=\{x \mid |x-1|<2\}$, then $A \cap B$ =____ .

3) The range of the function $f(x)=\begin{vmatrix} 2 & \cos x \\ \sin x & -1 \end{vmatrix}$ is ____ .

4) If $\vec{n}=(-2,1)$ is a normal of the straight line l， then the slope angle of *l* is (in form of an inverse trigonometric function).

5) In the binomial expansion of $(x-\frac{2}{x})^6$, the constant term is ____ .

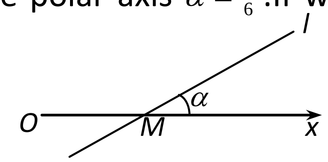
6) There are a series of cubes whose sides consist of a geometric sequence with 1 as its initial term and $\frac{1}{2}$ as its common ratio. If we denote their volumes as $V_1, V_2, \ldots, V_n, \ldots$， then $\lim_{n\to\infty}(V_1+V_2+\cdots+V_n)=$____ .

7) We define a function as $f(x)=e^{|x-a|}$ (*a* is constant).If $f(x)$ is increasing on $[1, +\infty)$， then the range of *a* is ____ .

8) If the side expansion plan of a cone is a semi-circle with area 2π, then the volume of the cone is _____ .

9) Assume $y = f(x) + x^2$ is an odd function and $f(1) = 1$.If $g(x) = f(x) + 2$， then $g(-1) =$____ .

10) See the graph， In the polar coordinate system, the straight line l passes through $M(2, 0)$ and form the angle with the polar axis $\alpha = \frac{\pi}{6}$.If we write the polar equation of l as $\rho = f(\theta)$，
then $f(\theta) =$_____ .

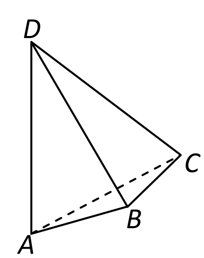


11) Three students participate in the high jump, long jump and shot projection competitions. If each student chooses two items, then the probability of exact two students who have chosen the same items is (give your result in the simplest fraction).

12)In Parallelogram *ABCD*， $\angle A=\frac{\pi}{3}$, the sides *AB* and *AD* are 2 and 1 respectively. If *M* and *N* are on sides *BC* and *CD* respectively satisfying $\frac{|\overrightarrow{BM}|}{|\overrightarrow{BC}|} = \frac{|\overrightarrow{CN}|}{|\overrightarrow{CD}|}$， then the range of $\overrightarrow{AM} \cdot \overrightarrow{AN}$ is _____ .

13) If the graph of the function $y = f(x)$ is the folded line *ABC*， with *A*(0,0)， *B*($\frac{1}{2}$,5)， *C*(1,0), then the area of the shape surrounded by the function $y = xf(x)\ (0 \le x \le 1)$ and the x-axis is ____ .

14)See the graph. *AD* and *BC* are two edges perpendicular to each other in the tetrahedron. *BC*=2. If *AD*=2*c* and *AB*+*BD*=*AC*+*CD*=2*a*， with *a*、*c* being constants， then the maximum volume of the tetrahedron is _____ .



2. Multiple choice (4 questions and 20 marks)

15) If $1+\sqrt{2}\,i$ is a complex root of the real-coefficient equation

$x^2+bx+c=0$, then ( )

(A) $b=2, c=3$. (B) $b=-2, c=3$. (C) $b=-2, c=-1$. (D) $b=2, c=-1$.

16) In the triangle ΔABC, if $\sin^2 A+\sin^2 B<\sin^2 C$,then ΔABC is ( )

(A) an acute triangle (B) a right-angled triangle (C) an obtuse triangle

(D) uncertain

17) Let $10\le x_1<x_2<x_3<x_4\le 10^4$ and $x_5=10^5$. The random variable ξ_1 takes values x_1, x_2, x_3, x_4 and x_5 with equal probability 0.2. The other random ξ_2 takes values $\frac{x_1+x_2}{2}$, $\frac{x_2+x_3}{2}$, $\frac{x_3+x_4}{2}$, $\frac{x_4+x_5}{2}$ and $\frac{x_5+x_1}{2}$ also with equal probability 0.2. If we denote $D\xi_1$ and $D\xi_2$ as the variance of ξ_1 and ξ_2, respectively, then ( )

(A) $D\xi_1>D\xi_2$. (B) $D\xi_1=D\xi_2$. (C) $D\xi_1<D\xi_2$.

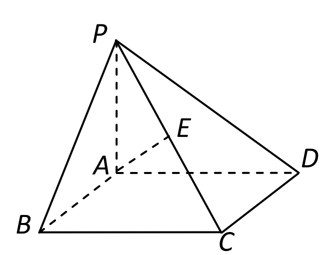
(D) the relationship of $D\xi_1$ and $D\xi_2$ depends on the values of x_1, x_2, x_3 and x_4.

18) If $a_n=\frac{1}{n}\sin\frac{n\pi}{25}$ and $S_n=a_1+a_2+\cdots+a_n$, then the number of positive numbers in $S_1,S_2,\cdots,S_{100}$ is ( )

(A) 25. (B) 50. (C) 75. (D) 100.

3. Solving problems (5 questions and 74 marks)

19) See the graph, in the Quadrangular pyramid *P-ABCD*, the base *ABCD* is a rectangular, *PA*$\perp$the base *ABCD*, *E* is the midpoint of *PC*. If *AB*=2, *AD*=2$\sqrt{2}$, *PA*=2.



Calculate

(1) the area of the triangle *PCD*; (6 marks)

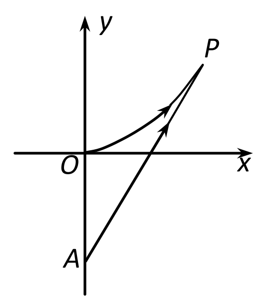
(2) the angle between the two straight lines *BC* and *AE*. (6 marks)

20) Define the function $f(x)=\lg(x+1)$.

(1) If $0<f(1-2x)-f(x)<1$, calculate the range of x; (6 marks)

(2) If $g(x)$ is a 2-periodic even function and $g(x)=f(x)$ when $0\le x\le 1$, calculate the inverse function of $y=g(x)$ $(x\in[1,2])$. (8 marks)

21) A boat to rescue is positioning a boat in wreckage: take the boat in wreckage as the origin and the north direction as y-axis



(1 mile as unit), the boat to rescue is at A which is 12 miles South from the boat in wreckage, see the graph.

If we assume:

- the boat in wreckage follows the path $y=\frac{12}{49}x^2$;
- the boat to rescue follows a straight line;
- after t hours, the x-coordinate of the boat in wreckage is $7t$.

Then

(a) If $t=0.5$, calculate the y-coordinate of P where the boat in wreckage is. If the two boats meet at P, calculate the speed of the boat to rescue(magnitude and direction); (6 marks)

(b) at least how fast should the boat to rescue travel to catch the boat in wreckage? (8 marks)

22) In the plane rectangular coordinate system xOy, we have a hyperbola

$C_1: 2x^2 - y^2 = 1$.

(1) through the left vertex of C_1 we draw a straight line parallel to one asymptote of C_1, calculate the area of the triangle surrounded by this line, the other asymptote and x-axis; (4 marks)

(2) Let a straight line with slope 1 intersect C_1 at *P* and *Q*, if *l* is tangential to the circle $x^2 + y^2 = 1$, prove： $OP \perp OQ$; (6 marks)

(3) Let the ellipse $C_2: 4x^2 + y^2 = 1$. If *M* and *N* are moving on C_1 and C_2, respectively， and $OM \perp ON$， prove : the distance from *O* the straight line is constant. (6 marks)

23) Let set $X = \{-1, x_1, x_2, \cdots, x_n\}$， with $0 < x_1 < x_2 < \cdots < x_n$， $n \geq 2$， define a vector set $Y = \{\vec{a} \mid \vec{a} = (s,t), s \in X, t \in X\}$.If for any $\vec{a_1} \in Y$,there exists $\vec{a_2} \in Y$, such that $\vec{a_1} \cdot \vec{a_2} = 0$, then we call *X* having the property **P**. For example, $X = \{-1, 1, 2\}$ has the property **P**

(1) If $x > 2$， and $\{-1, 1, 2, x\}$， calculate the value of *x*; (4 marks)

(2) If *X* has the property **P**, prove: $1 \in X$, and when $x_n > 1$， $x_1=1$; (6 marks)

(3) If *X* has the property **P**， and $x_1=1$， $x_2=q$ (*q* is constant), calculate the general term formula of the sequence $x_1, x_2, \cdots, x_n$. (8 marks)

NB: The original paper and answers (in Chinese) are available at

http://learning.sohu.com/20120609/n345172111.shtml (last accessed May 2013)

Appendix 4: General Baccalaureate (France)

Baccalauréat Général

(General Baccalaureate)

Session 2012

Mathematics

Stream: sciences

Duration of the test: 4 hours   Weighting: 9

TEACHING SPECIALIST

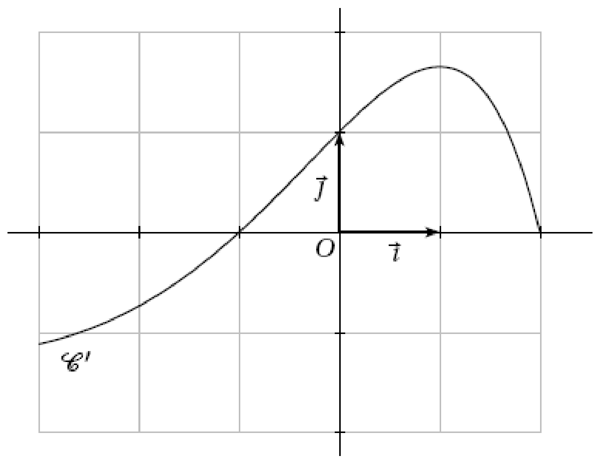
Electronic pocket calculators are permitted, in accordance with the regulations.

Exercise 1 (4 marks)

Common to all candidates

The plane is equipped with an orthonormal (O; $\vec{\imath}$, $\vec{\jmath}$). Consider a function differentiable f on the interval [-3,2]. We have the following information:

- f (0) = −1.
- The derivative f' of the function f has the following graph



For each of the following statements, say whether it is true or false and justify the answer.

1. For all x in the interval [-3, -1], $f'(x) \leq 0$.
2. The function f is increasing on the interval [-1,2].
3. For all x in the interval [-3, 2], $f(x) \geq -1$.
4. Assume 𝒞 is the graph of the function f. The tangent to the curve 𝒞 at x=0 passes through the point (1, 0).

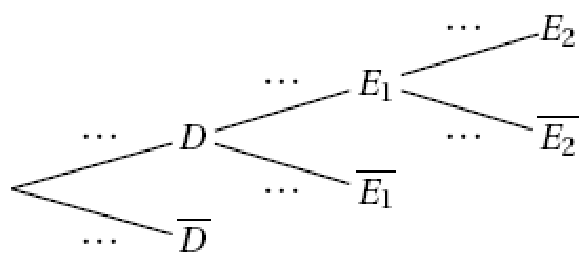
Exercise 2 (5 marks)

Common to all candidates

To hire its managers a company uses a recruitment firm. The procedure used is as follows. The firm makes an initial selection of candidates on file. 40% of applications received are validated and sent to the company. The candidates thus selected pass an initial interview, after which 70% of them are retained. These are invited to a final interview with the director of human resources who recruit 25% candidates met.

1. We choose the case of a candidate at random. The following events are considered:
   D: « The candidate is held on file »,
   E_1: « The candidate is held at the end of the first interview»,
   E_2: « The candidate is recruited ».

   a. Reproduce and complete the weighted tree below.



   b. Calculate the probability of the event E_1.

   c. Let F be the event «The candidate is not hired». Demonstrate that the probability of the event F is equal to 0.93.

2. Five friends apply for the management job in this company. Their application cases are independent of each other. It is assumed that the probability of each of them is recruited is equal to 0.07. We denote by X the random variable giving the number of people recruited from these five candidates.

a. Prove that X has a binomial distribution and specify the parameters of the law.

b. Calculate the probability that exactly two of the five friends are recruited. we round up to 10^{-3}.

3. At least how many files that the recruitment firm must treat so that the probability of hiring at least a candidate is greater than 0.999?

Exercise 3 (6 marks)

Common to all candidates

It is possible to do Part C without addressing Part B.

Part A

We denote by f the function defined on the interval $[1, +\infty)$ by

$$f(x) = \frac{1}{x+1} + \ln\left(\frac{x}{x+1}\right)$$

1. Determine the limit of the function f when $x \to +\infty$.

2. Prove that for all x in the interval $[1, +\infty)$, $f'(x) = \frac{1}{x(x+1)^2}$.
3. Deduce the sign of the function f on the interval $[1, +\infty)$.

Part B

Let (u_n) be the sequence defined for all positive integers by $u_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n)$.

1. We consider the following algorithm:

Variables:	i and n are natural numbers. u is a real number.
Input:	Ask the user for the value of n.
Initialization:	Assign the value 0 to u.
Iteration:	For i varying from 1 to n, assign to u the value $u + \frac{1}{i}$.
Output:	Show u.

Give the exact value displayed by this algorithm when the user enters the value n = 3.

2. Copy and complete the previous algorithm so that it displays the value of u_n when the user enters the value of n.

3. Here are the results from the modified algorithm, rounded to 10^{-3}.

n	4	5	6	7	8	9	10	100	1000	1500	2000
U_n	0.697	0.674	0.658	0.647	0.638	0.632	0.626	0.582	0.578	0.578	0.577

Using this table, make conjectures about the direction of change of the sequence (u_n) and its eventual convergence.

Part C

This part can be processed independently of Part B.

It helps demonstrate the conjectures made about the sequence (u_n) such that for any positive integer n,

$$u_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n).$$

1. Prove that for any positive integer n,

$$u_{n+1} - u_n = f(n)$$

where f is the function defined in Part A. Deduce the direction of variation of the sequence u_n.

2. a. Let k be a strictly positive integer. Justify the inequality

$$\int_k^{k+1} \left(\frac{1}{k} - \frac{1}{x}\right) dx \geq 0.$$

Deduce that

$$\int_k^{k+1} \frac{1}{x} dx \leq \frac{1}{k}.$$

Prove the inequality

$$\ln(k+1) - \ln(k) \leq \frac{1}{k} \qquad (1)$$

b. Write the inequality (1) by successively replacing k by 1, 2, ..., n and show that for any positive integer n,

$$\ln(n+1) \leq 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}.$$

c. Deduce that for any positive integer n, $u_n \geq 0$.

3. Prove that the sequence (u_n) is convergent. We do not ask to calculate the limit.

Exercise 4 (5 marks)

Candidates having followed teaching specialty

The complex plane is equipped with an orthonormal $(O;\ \vec{u},\ \vec{v})$.

We denote by A, B and C the points respectively associated to $z_A = -1 + i$, $z_B = 2i$ and $z_C = 1 + 3i$ and $\mathcal{D}$ the line $y = x + 2$.

1. Prove that the points A, B and C belong to the line $\mathcal{D}$.
   On the figure that is made by taking 2 cm graphics unit, place the points A, B, C, and draw the line $\mathcal{D}$.

2. Solve the equation $(1 + i)z + 3 - i = 0$ and check that the solution of this equation is associated to a point that does not belong to the line $\mathcal{D}$.

In the following exercise, we call f the function that, at any point M associated to z different from -1 +2 i, match point M′ associated to $\frac{1}{(1+i)z+3-i}$.

The purpose of the exercise is to determine the image of the line $\mathcal{D}$ under f.

3. Let g the plane transformation that, for all point M associated to z, match M_1 associated to $(1 + i)z + 3 - i$.

   a. Determine the nature and characteristic features of the transformation g.

   b. Calculate the numbers related to points A_1, B_1 and C_1, respective images of the points A, B and C under g.

   c. Determine the image $\mathcal{D}_1$ of $\mathcal{D}$ under the transformation g and draw the figure.
4. h is the function that, at any point M associated to nonzero z, match to point M_2 associated to $\frac{1}{z}$.

a. Determine the associated points $h(A_1)$, $h(B_1)$ and $h(C_1)$, and place these points on the figure.

b. Prove that for any nonzero complex number z, we have:

$$\left|\frac{1}{z} - \frac{1}{2}\right| = \frac{1}{2} \Leftrightarrow |z - 2| = |z|$$

c. Deduce that the image of line $\mathcal{D}_1$ under h is included in a circle $\mathscr{C}$ with center and radius be specified. Draw the circle in the figure.

d. Demonstrate that any point on the circle $\mathscr{C}$, which is distinct from O is the image of a point on the straight line $\mathcal{D}_1$ under h.

5. Determine the image of the line $\mathcal{D}$ under f.

NB: The original paper (in French) is available at

http://www.maths-france.fr/Terminale/TerminaleS/ProblemesBac/2012/BacS_Juin2012_Specialite_Enonce.pdf (last accessed May 2013)

and the answers (in French) is available at

http://www.maths-france.fr/Terminale/TerminaleS/ProblemesBac/2012/BacS_Juin2012_Specialite_Corrige.pdf (last accessed May 2013)

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